Addressing Fractional Dimensionality in the Application of Weather Index Insurance and Climate Risk Financing in Agricultural Development: A Dynamic Triggering Approach

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Abstract

Climate risk financing programs in agriculture have caught the attention of researchers and policy makers over the last decade. Weather index insurance has emerged as a promising market-based risk financing mechanism. However, to develop a suitable weather index insurance mechanism it is essential to incorporate the distribution of underlying weather and climate risks to a specific event model that can minimize intra-seasonal basis risk. In this paper we investigate the erratic nature of rainfall patterns in Kenya using CHIRPS rainfall data from 1983-2017. We find that the patterns of rainfall are fractional, both erratic and persistent which is consistent with the Noah and Joseph effects well known in mathematics. The erratic nature of rainfall emerges from the breakdown of the convergence to a normal distribution. Instead we find that the distribution about the average is approximately lognormal, with an almost 50% higher chance of deficit rainfall below the mean versus adequate rainfall above the mean. We find that the rainfall patterns obey Hurst law and the measured Hurst coefficients for seasonal rainfall pattern across all years range from a low of 0.137 to a high above 0.685. To incorporate the erratic and persistent nature of seasonal rainfall, we develop a new approach to weather index insurance based upon the accumulated rainfall in any 21-day period falling below 60% of the long-term average for that same 21-day period. We argue that this approach is more satisfactory to matching drought conditions within and between various phenological stages of growth.

Key words: Fractional Weather, Insurance-Linked Credit, Joseph Effect, Agricultural weather risk, Phenological growth, Hurst coefficient

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1. Introduction

All too often, scholars and practitioners examining weather data for the purpose of developing weather index insurance (WII) make the assumption, or presumption, that the data are Gaussian and Markovian, meaning that day over day or season over season measures are independent and uncorrelated. For example, index insurance design presented in Mahul and Skees (2007) for livestock insurance in Mongolia, Khalil et al. (2007) for El Nino insurance in Peru, Makaudze and Miranda (2010) for drought insurance in Zimbabwe, Chantarat et al. (2013) and Woodard et al. (2016) for livestock insurance in Kenya used seasonal accumulation of weather indexes for pricing insurance and assumed independent relationship between sequential timeseries weather data. This may not be the wisest approach. When meteorologists speak of weather 'patterns' they are not making references to sequences of independent events, but events of some duration that in one way or another are correlated. This in itself should be warning enough that weather patterns do not follow the Gaussian variance law $VAR\left[x_{t+S}-x_t\right]=VAR\left[x_t-x_{t-1}\right]S^{\frac{1}{2}}$ but rather a form of Hurst's Law, $VAR\left[x_{t+S}-x_t\right]=VAR\left[x_t-x_{t-1}\right]S^{2H}$ where H, referred to as Hurst's coefficient, is a non-arbitrary constant (Hurst 1951). As will be shown presently, H, captures persistence in negative (H<0.5) and positive (H>0.5) correlations across the time domain. Only for H=0.5 does the Gauss-Markov assumption hold. The implication of this for climate-based weather index insurance is evident. Should the Gauss-Markov assumption be used in the pricing of WII it will overestimate the variance for H<0.5 and underestimate the variance for H>0.5. The direct corollary is that the actuarial value of WII will be too high for H<0.5 and too low for H>0.5.

Fractional dimensionality of weather events should be an important consideration for capturing weather extremities. In Hurst's (1951) original work on Hydrology he was examining the water flow of the Nile to engineer dams. These dams had to withhold the extremes of water flow. In Hurst's rescaled-range approach (which is similar to but different from our Brownian approach) he discovered that the flow of water varied proportionately to d^H with $0.5 < H < 1$, and not $d^{0.5}$ as had previously been thought (Mandelbrot and Van Ness, 1968). With coefficients of $H \approx 0.7$ the upper-bound extremities of water flow were much higher than expected, and consequently the dams had to be engineered to withhold low-frequency, extreme events that exceeded the flows observed to that date. One can imagine now if the variable d were otherwise interpreted as cumulative rainfall and followed a d^H law then the actuarial measures of extreme rainfall would need to account for the fractional extremities, even if those extremes had not previously been observed. We believe these considerations to be important in the development of a sustainable weather index insurance product and explore them in this paper in the context of an operational insurance-linked credit in Kenya.

In Mandelbrot and Wallis's classical 1968 exploration into the fractional properties of water flow, they engagingly introduce the problem with biblical references to the 'Noah" effect, which refers to the fact that precipitations can have very sudden disruptions, and the 'Joseph Effect' in which precipitations in a period can have correlation with precipitations in the distant past. The extremities of rainfall are bounded from below at zero and are theoretically limitless from above, although not unlikely without bound. To what extent either the Noah Effect or the Joseph Effect impacts the human or natural condition is determined by the mathematical mix of frequency, duration and intensity and with respect to the ecology under consideration.

In the context of agricultural weather insurance – the broader subject matter of this paper – these climate considerations are important. The context of this paper is the design of a weather index insurance product that can be imbedded into a credit product to manage drought related agricultural risk as well as to provide access to credit to smallholder farmers. In this way we make two advances to the literature and practice in the design of WII and the application to climate risk financing. On this latter point, insurance linked or bundled credit products have been promoted by a number of scholars including Skees and Barnett (2006), Gine and Yang (2009), Carter et al. (2011), Karlan et al. (2011), Miranda and Gonzales-Vega (2011), Collier et al. (2011), Shee and Turvey (2012), Shee et al. (2015), Marr et al. (2016), Pelka et al (2015), Carter et al. (2016), von Negenborn et al. (2018) and Mishra et al. (2018). Clarke and Grenham (2013) and Jensen et al. (2016) point out that basis risk limits farmers' investment in weather index insurance. In a recent article, Muller et al. (2017) argue the need of careful consideration of local socio-ecological context when developing an agricultural risk transfer product. Insurance linked credit mechanism responds to several of the critiques offered by Binswanger-Mkhize (2012) who argues that until the problems of basis risk and credit access can be resolved the advancements of various forms of weather insurance as a promising mechanism to transfer risk from poor farmers to the insurance markets on a large scale might be overstated. Although the promise of index insurance in alleviating poverty traps has been widely discussed (e.g. Barrett et al. 2007), and some successes reported (e.g. Chantarat et al. 2017), the Binswanger-Mkhize critique should not be treated lightly.

In the fall of 2017 our research group piloted an insurance linked credit product in Machakos County in Kenya, with loan indemnities linked to long and short rains (Shee et al. 2019). Our study area comprises five Sub-Counties in Machakos County and is spread over 13 locations as specified in Figure 1. This is a semi-arid and hilly terrain area that receives very low annual rainfall of around 700 mm per year with average rainfall in long and short rain seasons being 315 and 266 mm, respectively (GOK 2014). Due to this semi-arid climate, agriculture is practiced by smallholder farmers with maize being the main food crop.

[Insert Figure 1]

In our efforts, it became abundantly clear that standard approaches to weather index insurance based on seasonal cumulative rainfall were inadequate in measuring within-season risk. To address the phenological problem we develop a rainfall insurance based on what we refer to as a dynamic trigger. This trigger establishes an indemnity if the accumulated rainfall in any 21-day period is below 60% of the historical average rainfall in that same 21-day period for a given year. This will be explored in more depth later, but when we examined the 'average' path of overlapping 21-day measures and took the deviation of each year's equivalent measure we found the distribution of the difference to be non-normal. Indeed, we find it to be close to a lognormal distribution with the probability that below-normal rainfall in our study region was approximately 50% more likely than above normal rainfall. The failure of normality suggests also a failure in the Gauss-Markov assumption normally assumed in a first-guess approach to statistical assumption. But this also comes with a possible failure in the independence assumption and Brownian presumption of the historical time-path of our data series – as limited as it is. As Mandelbrot and Wallis (1968) point out the failure to recognize the non-Markov possibilities would greatly underestimate the duration and intensity of the longest drought.

We explore fractional weather pattern by measuring Hurst coefficients within the Mandelbrot-Wallis framework to investigate non-Markov possibilities. Our analysis of fractional weather patterns will encourage researchers who develop or evaluate the many varieties of weather index insurance models to treat with greater seriousness the combined Noah and Joseph effects (sudden reversal and long persistence in weather pattern). Of course, weather patterns are mixed in terms of frequency, duration, and intensity, but one clear implication of our investigation is that fractional patterns within an agricultural growing season should not be ignored, and that within-season WII should be designed with multiple events that traverse the various stages of phenological growth. Our approach to dynamic triggering of multiple events in designing agricultural WII is an important contribution of this paper to incorporate fractionality of weather and to reduce intertemporal basis risk in WII.

The next section discusses the literature on specific events and phenological growth in weather index insurance. Next, we develop with greater granularity the statistical genesis of the Hurst coefficient within the Mandelbrot-Wallis framework. This is followed by a description of the fractional patterning of rainfall in Machakos where we provide Hurst measures. We then provide a possible solution to fractional rainfall patterns by providing a new structure for within-season WII based on a dynamic trigger approach that traverses phenological growth stages to provide up to four indemnifiable events across the growing season. Section 6 concludes with broader policy implications.

2. Specific events and phenological growth in weather index insurance

Turvey (2001) notes that specific weather events can be linked to insurance coverage against crop production, which is usually affected by phenological growth dependent on weather conditions such as rainfall, temperature, and soil moisture. Norton et al. (2016) for example, suggest that weather insurance ought not to be viewed as a perfect substitute for multiple peril crop insurance, but as a risk-transfer instrument that should be specifically targeted towards covariate events that are weather-sensitive such as Karnal Bunt and Stewart's disease. Particular weather conditions can also give rise to insect infestations for which Richards et al. (2006) suggest 'bug options'.

The weakness in the application of weather insurance in both developing and developed agricultural economies is the existence of 'basis' risk. Generally speaking, basis risk, refers to the observed variability in an insured asset that is seemingly uncorrelated, or weakly correlated, with a proxy measure of risk. Basis risk can arise spatially by distance, altitude and geomorphology

(Norton et al., 2012; Heimfarth et al., 2012; Heimfarth and Musshoff, 2011; Woodard and Garcia 2008). However, it is increasingly being recognized that basis risk also arises with the patterning of weather within a season at a specific location. Intra-seasonal basis risk deals with the failure of a specified index to adequately capture within-season variability, leading to excessive type I or type II errors. Type I error refers to insurance payments when no crop damage is observed, while type II error refers to situations in which crop damage is observed but no indemnity is paid. In a more general way we can say that added specificity to the weather index ought to reduce both Type I and Type II error, at least in probabilistic terms. One approach to doing this is to recognize that rather than a single index for an entire season (e.g. based on cumulative rainfall), the growing season can be broken up into multiple events.

An obvious starting point for multiple events is to examine the phenology of crop production. In general, there are three stages of crop growth; the vegetative stage from germination to panicle initiation, the reproduction phase from panicle formation to flowering, and the ripening phase from flowering to the final formation of grain. Conradt, Ginger and Sporri (2015), Dalhaus and Finger (2016), Dalhaus, Musshoff and Finger (2018), and Shi and Jiang (2016) have all explored models for including phenology in an index for weather insurance. Shi and Jiang (2016) created a composite index based on sub-period weather data that covers the three stages of growth. Using rice in China, the vegetative stage covered seedling, tillering and stem elongation (31 days + 20 days + 20 days = 71 days), the reproductive phase included panicle formation through flowering (33 days + 8 days = 41 days), and the ripening phase that covered an additional 57 days. All told, the insurable season covered 169 days. Using a two-stage procedure they mapped discrete-time days-in-season for each phase, and continuous time observations on rainfall, relative humidity, sunshine and temperature to construct a parametric composite index.

Dalhause and Finger (2016) broke the growing season into multiple parts and using historical and observational data on crop phenology for German wheat, designed multiple event index insurance for which the farmer could choose amongst its ‘calendar’ parts. They find that use of phenological observations significantly reduced basis risk. Likewise, Conradt et al. (2015) investigated a flexible index insurance plan for Growing Degree Days in Khazakstan. ‘Flexibility’ in their context was to determine the beginning and end periods of the phenological growth stages (start and end dates) which vary from year to year. The advantage to weather risk management is the recognition that within season weather patterns are not constant from year to year and will generally have different start and end dates from one year to the next. Depending upon the start-end date signals the days over which insurance is to be calculated. In terms of phenological growth, seeds will germinate much sooner in warm years than cold years, but if the dates for index measurement were fixed the index might weigh more heavily the effects of a cool year versus a warm year. A similar approach was deployed by Dalhaus et al. (2018) for German wheat, finding that developing weather index insurance using published phenological observations increased farmers’ utility and reduced financial exposure to drought risk. The approach was to establish start and end dates of each growth stage using growing degree days, and then accumulating rainfall within each stage to develop and indemnity structure.

These studies raise a certain number of issues for weather index insurance. Most critical is the date of growing season onset. Maize is the dominant food crop grown in our study area where the reference date for the beginning of long rains is October 15th. From the project baseline household survey, we found that some farmers seed early in case the rains come early, while others withhold seeding until the rain has observably arrived. Both are rationally precautionary. However, if farmers spread seeding across a two-week period defining risks according to specific calendar dates is a wobbly venture. The targeting of specific events as prescribed in Turvey (2001) would be effective only if planting and weather conditions were relatively homogenous across farms in a particular heat unit isocline. But variance in weather conditions – heat and rainfall – can shift the stages across calendar date boundaries, rendering the ideal of specificity benign. A possible approach would be to widen the date range so that in probability the shifting patterns of weather affecting a particular stage of growth, for example silking in corn, would adequately be captured. From this point of view, basing weather index insurance on broader phenological stages may be a sensible approach to balancing Type I and Type II error.

Even so, it is troubling that the patterning of weather variables should be confined to proximal calendar dates of the various growth stages as if each stage can be treated as an individual and independent peril. While clearly adequate for reducing within-season basis risk, the approach does not consider overlapping perils. For example, if a late vegetative stage drought overlaps or spans the next stage tillering it is entirely possible that neither event would trigger a payout yet crop damage would be measurable. Turvey (2001) and Turvey and Norton (2008) address this problem in their specific event approach. In Turvey's (2001) Ontario study, an insurance would payout if the rainfall in any non-overlapping 14-day period (an event) was equal to zero, making up to 4 events between June 1 and July 31. Turvey and Norton (2008, Figure 5A) defined an event when the 21 days cumulative rainfall fell below 1 inch, making up to 3 events in the season.

The advantage of a specific event approach to measuring weather risk is that by slicing the growing season into fixed day events that are overlapping in measurement, but not overlapping on indemnity, is the added flexibility to capture risks within each phenological stage, but also across the temporal boundaries of the phenological stages. This would further reduce the potential for Type II error, as well as Type I error. However, the specifications discussed in Turvey (2001) and

Turvey and Norton (2008) are also imperfect in the sense that the nature of the event is assumed to be constant for all events, i.e. rainfall less than 1 inch in 21 days. In reality, the seasonal patterning of rainfall should not be assumed constant, nor should the triggering event. Indeed, cropping systems evolve according to the historical weather patterns that define the local ecology and growing conditions. It will be an exaggeration, for example, to treat equally Ardmore Oklahoma – the center point of the 1930's dust bowl – with Ithaca NY to its northeast. From Turvey and Norton (2008) Ardmore averaged 9.08 inches of rainfall between June 1 and August 31, while Ithaca averaged 10.74 inches. A specific event metric that would pay out if rainfall fell below 5 inches over this period would have paid out nearly once in every 5 years (21.21%) in Ardmore, but would never pay payout in Ithaca; Likewise a specific event defined by 7 straight days with daily temperatures exceeding 90F would pay out once in only 3 of 100 years in Ithaca, but would pay out on 4 or more distinct non-overlapping events in nearly 93 of every 100 years (92.71%) in Ardmore. While these exemplify the spatial differences in the large, Norton et al. (2012) illustrate how basis risk correlates with differences in distance, altitude, and direction in longitude and latitude from a given weather station.

3. Methodological framework for fractional dimensionality

a. Scaling properties and erratic weather processes

To better understand within-season weather patterns in the context of designing weather index insurance this section and what follows examine fractional dimensionality of weather including fractional Brownian processes and the Hurst measure. We start with Mandelbrot and Wallis (1968) who point to three characterizations of stochastic processes that might give rise to a Brownian Gauss-Markov process. The first is that some process $x(t)$ will satisfy the law of large numbers in the sense that its expected value tends to a limit, $E\left[x(T)\right]$ as T approaches infinite. The second is with respect of the central limit theorem in that for large T the distribution around the average becomes approximately Gaussian for T to infinite. The third deals with the scaling properties of the process and independent increments.

When one or more of these conditions are violated, Mandelbrot and Wallis (1968) refer to the processes as being ‘erratic’. Thus ‘Joseph-erratic’ might refer to a phenomenon of an extraordinary term of wetness or dryness within a time span such that localized path dependence and measurable correlations are not obscured or mitigated by the law of large numbers. ‘Noah-erratic’ behavior occurs when the intensity of the weather event (precipitation or lack thereof) is so great as to affect the average of the measured event for many periods (e.g. years) after the event occur. Notably, both Joseph- and Noah-erratic behavior can occur simultaneously, and feed off each other.

A first approach to considering Noah and Joseph effects is to assume that the weather patterns of interest follow a fractional Brownian motion. Mandelbrot and Van Ness (1968) detail the properties of fractional Brownian motion with considerable depth. We take a different approach to get the main points across. To get a sense of the scaling properties that might give rise to erratic behavior, consider the ordered set of measures (e.g. precipitation) $X_T = \{x_{t+S}, x_{t+S-1}, x_{t+S-2}, ..., x_t\}$. For convenience of illustration set $t = 0$ and $S = 100$. Then, the difference $x_{100} - x_0$ can be expressed equivalently as

(1) $x_{100} - x_0 = x_{100} + (x_{99} - x_{99}) + (x_{98} - x_{98}) + ... + (x_2 - x_2) + (x_1 - x_1) - x_0$

or

(2) $x_{100} - x_0 = (x_{100} - x_{99}) + (x_{99} - x_{98}) + ... + (x_2 - x_1) + (x_1 - x_0)$

Assuming equivalence of any $\left(x_t - x_{t-1}\right)$

$$
(3)\quad \begin{aligned} E\left[x_{100}-x_0\right] &= E\left[\left(x_{100}-x_{99}\right)+\left(x_{99}-x_{98}\right)+\ldots+\left(x_2-x_1\right)+\left(x_1-x_0\right)\right] \\ &= E\left(x_{100}-x_{99}\right)+E\left(x_{99}-x_{98}\right)+\ldots+E\left(x_2-x_1\right)+E\left(x_1-x_0\right) \\ &= 100E\left(x_1-x_0\right) \end{aligned}
$$

Or more generally

$$
(4)\quad E\left[x_{t+S}-x_t\right]=SE\left(x_1-x_0\right)
$$

The variance of the differences can be expressed as

$$
(5)\quad VAR\left[x_{t+S}-x_t\right]=\sum_{t=1}^{S} E\left[\left(x_t-x_{t-1}\right)-E\left(x_t-x_{t-1}\right)\right]^2+COV\left[x_{t+S}-x_t\right],
$$

where

$$
(6)\quad COV\left[x_{t+S}-x_t\right]=2\sum_{(i\in t)=1}^{S}\sum_{(j\in t)=1}^{S} E\left[\left(x_i-x_{i-1}\right)-E\left(x_i-x_{i-1}\right)\right]\left[\left(x_j-x_{j-1}\right)-E\left(x_j-x_{j-1}\right)\right], i\neq j
$$

It is from this measure of variance that the scaling properties related to erratic behavior arise. The scaling can be defined by S^{2H}, where H is the Hurst coefficient. The Hurst coefficient plays a crucial role in the identification of fractional properties of time series. We can restate variance in terms of the Hurst scaling rule as,

$$
(7)\quad VAR\left[x_{t+S}-x_t\right]=VAR\left[x_t-x_{t-1}\right]S^{2H}.
$$

b. Self-affine and self-similar processes

Equation (7) holds a particular meaning to the scaling properties of Brownian motion. Self-affine refers to an invariance with respect to time scale. If $x\left(t,w\right)$ has self-affine increments with parameter $0\leq h\leq 1$, then for a particular Brownian function $B\left(t_0+s,w\right)\triangleq h^{-H}B\left(t_0+hs,w\right)$ where

$\triangleq$ means that the two sides have the same finite joint distribution and one drawn from the same space. Thus, rewriting (7) as $\sigma^2_{t_0+S} = \sigma^2_{t_0} S^{2H}$, and after scaling by h, $\sigma^2_{t_0+hS} = \sigma^2_{t_0} h^{2H} S^{2H}$ or $\sigma_{t_0+hS} = \sigma_{t_0} h^H S^H$, then rescaling by h^{-H} returns (7).

Self-similarity is a related property. It states that the rescaled function has the same distribution for every T. Again, rewriting (7) as a variance ratio, $\frac{\sigma^2_{t_0+S}}{\sigma^2_{t_0}} = S^{2H}$, or $\frac{Log\left(\frac{\sigma^2_{t_0+S}}{\sigma^2_{t_0}}\right)}{Log(S)} = 2H$ so that for any particular change in the time step (e.g. S), the variance will increase at the fractional dimension rate, $2H$.

Moreover, since there is no reason to believe a priori that $VAR\left[x_t - x_{t-1}\right] \neq VAR\left[x_{t+1} - x_t\right]$ we can restate the scaled variance measure as

(8) $VAR\left[x_t - x_{t-1}\right] S^{2H} = VAR\left[x_t - x_{t-1}\right] S + COV\left[x_{t+S} - x_t\right]$,

and from this we restate covariance as,

(9) $COV\left[x_{t+S} - x_t\right] = VAR\left[x_t - x_{t-1}\right]\left(S^{2H} - S\right)$.

Expressed in this way the covariance term defines the Brownian set and the nature of dependence in fractional processes. The relationships are as follows;

$$(10) \qquad S \Leftarrow \left\{ \begin{array}{l} H = \frac{1}{2} \rightarrow COV\left[x_{t+S} - x_t\right] = 0,\ S = \frac{VAR\left[x_{t+S} - x_t\right]}{VAR\left[x_t - x_{t-1}\right]} \\ H > \frac{1}{2} \rightarrow COV\left[x_{t+S} - x_t\right] > 0,\ S > \frac{VAR\left[x_{t+S} - x_t\right]}{VAR\left[x_t - x_{t-1}\right]} \\ H < \frac{1}{2} \rightarrow COV\left[x_{t+S} - x_t\right] < 0,\ S < \frac{VAR\left[x_{t+S} - x_t\right]}{VAR\left[x_t - x_{t-1}\right]} \end{array} \right\}.$$

First, for $H = \frac{1}{2}$, the covariance term collapses to zero. In this state there is no intertemporal correlation between changes in the measure. This defines a pure Gauss-Markov, memoryless process of a standard Brownian motion and the linear-in-variance assumption. Here, the variance of the measure over a hundred days, or months, or years is 100 times the 1-step measure in days or months or years. And it appears to be predictable. This condition will satisfy all three of the characteristics identified by Mandelbrot and Wallis (1968) and is consistent with the usual interpretation of a random walk. However, for $H < \frac{1}{2}$ and $H > \frac{1}{2}$ the scaling properties are not memoryless, and thus violate the Markov property. For $H > \frac{1}{2}$, systemic positive correlation compounds the variance so that the variance of the measure over 100 days, or months, or years will be greater than 100 times the 1-step measure. It is persistent. Likewise, for $H < \frac{1}{2}$, the covariance will be systemically negative so that an increase in the measure of some time scale will, in probability, reverse itself in a mean-reverting or ergodic way. The variance of the measure over 100 days, or months, or years, will be less than 100 times the variance of the 1-step measure.

When $H \neq \frac{1}{2}$ the third condition is violated and this in return gives rise to Noah- and Joseph-erratic behavior. Whether variance is expanding or contracting in scale, the process becomes far less predictable. When the process is erratic-persistent the precipitation patterns are subject to longer excursions (an excursion is a measure of the length of a stochastic process on the space of paths) in wetness and dryness; when the process is ergodic-erratic the patterns of rainfall are more oscillatory with increasingly shorter excursion paths as H gets smaller.

It becomes evident then that Hurst's law has something important to add to the broader discussion of weather index insurance. We will show presently for the case of Machakos County, Kenya in Sub-Saharan Africa that indeed Hurst's law holds and in doing so violates Mandelbrot and Wallis's (1968) third condition; but equally important is our finding that the second condition- that of Gaussian-normal error around the mean path of our rainfall measure - also fails.

c. Excursion patterns

A final consideration that merits attention for WII are excursion paths (Ito 1972, 2007; Rogers 1989; Pitman and Yor 2007). In the most general way any Brownian motion is a continuous-time process with a recurrent state. By 'recurrent state' it is meant that at some unknown, and random time in the future, the stochastic path will return to its original state or a fixed point. The Joseph effect, for example is described by two long excursions, the first an excursion of 7 years in which conditions are good, followed by 7 years in which conditions are bad. If we use the inflection point between good and bad years as a fixed-point barrier, an excursion begins and ends as it crosses this point from above or below.

There are two measures of excursion. The first referred to as 'local time' that counts the number of times the path crosses the barrier in a fixed amount of time. The second measure is the length of the excursion, referred to as 'stopping time', τ , which is measured by the interval between barrier crossings. These intervals are not of fixed length as the biblical reference to Joseph suggests, but are random. Ito has shown that the excursion point process follows a Poisson distribution, $f\left(\tau\right)=\frac{\lambda^{\tau}e^{-\lambda}}{\tau!}$, with expected value $E\left[\tau\right]=\lambda$, and skewness, $Skew=\frac{1}{\sqrt{\lambda}}$

We assert that the relationship between mean stopping time λ and H is described by a power law of the form $\lambda=\alpha H^{\beta}$, and by substitution $Skew=\frac{H^{-\frac{\beta}{2}}}{\sqrt{\alpha}}$. Differentiating the expected stopping time and skewness yields $\frac{\partial E\left[\tau\right]}{\partial H}=\alpha\beta H^{\beta-1}>0$ and $\frac{\partial Skew}{\partial H}=-\frac{\beta H^{-\left(1+\frac{\beta}{2}\right)}}{2\sqrt{\alpha}}<0$ respectively. Thus, as H increases, the expected stopping time increases non-linearly, but perhaps more important, skewness falls. In other words, for low Hurst the distribution of stopping times will be left modal and positively skewed, but as H increases the stopping time increases, and the distribution trends towards right modal and negative skew.

We see the implications for the design of a WII as follows: As weather patterns move towards a high Hurst state the weather conditions will become more persistent and longer in duration. If it is rainfall, and the excursion is above the barrier then rain will continue accumulating. This may be ideal for some values of H, but for higher values the longer the rainfall the closer one gets to the Noah effect, with significant flooding. Likewise, if rainfall is decreasing below the barrier, the higher Hurst values will increase the duration of drought conditions, and with higher frequency.

Conversely, low Hurst values have much smaller excursions with frequent reversals. For low Hurst a short period of rain will be followed by a short period with no, or little rain. These states may in fact be adequate to avoid drought conditions if the stopping times for rainfall, by chance, exceed the stopping times with low or no rainfall, but drought conditions can arise if the opposite were true and the stopping times of no or low rainfall, by chance, exceeded the stopping times with adequate rainfall.

4. Data and application

a. Fractional patterning of rainfall in study area

Our study area, Machakos County is a semi-arid and hilly terrain in Eastern province of Kenya where we are implementing a randomized control trial (RCT) to investigate bundled, or risk-contingent, credit based on long and short rains (for details on risk-contingent credit see Shee and Turvey 2012; Shee et al. 2015; Shee et al. 2019). It is generally agreed that the long rains start at October 15th and end on January 15th (GOK 2014; Shee et al. 2019). We analyze Climate Hazards Group InfraRed Precipitation with Station Data (CHIRPS) (Funk et al. 2015) supported by National Aeronautics and Space Administration (NASA) and National Oceanic and Atmospheric Administration (NOAA). Daily CHIRPS rainfall (satellite validated with station data) data from 1983 to January 2018 are extracted from http://chg.geog.ucsb.edu/data/chirps/. Typically, rainfall is not evenly distributed in a season but starts low, rises to a mid-season peak and then diminishes thereafter. In fact, the pattern appears to be uniformly described by a 6th order polynomial across all the sub-counties we examined. A typical pattern is illustrated in Figure 2, for Central Machakos, for the average of the 21-day overlapping cumulative rainfall from 1983 to 2017. Our use of a 21-day cumulative metric serves two purposes. First by taking a cumulative measure there is some smoothing of asynchronous rainfall patterns when it does not rain for multiple days. Second, our view is that WII is better focused on the extremes. Following Turvey (2001) and Turvey and Norton (2008) we define specific events to be 21 fixed days that are overlapping in measurement but non-overlapping in indemnity. A shorter time scale, say 14 days, is not uncommon, and perhaps too common for a WII product that needs to sustainably balance coverage, indemnity and premium.

[Insert Figure 2]

Figure 3 illustrates the day-to-day probability distribution of the deviation of the recorded weather pattern from the mean (N=2478). A typical assumption of randomness is that these deviations be normally distributed, however we find that the distribution is more closely aligned with the log normal distribution. The mean of the distribution is zero, as expected, but the skewness is 1.0123 with kurtosis of 4.178. The modal value of -24.86 is negative, and the probability that a deviation is negative is 59.8% against 40.2% chance of a positive deviation. In other words, if 21-day cumulative rainfall is to deviate from the long run mean, it is 50% more likely to be a negative deviation than a positive deviation.

[Insert Figure 3]

To investigate the properties of the distribution, we compute the Hurst coefficient for each year using the scaled variance method. Keeping in mind that within each year the long rain season comprised of only 93 days. Our Hurst measures are based on overlapping 21-day measures, so N=72. Comparatively, this is a small number for estimating H, so we should not be surprised by a wide variation in estimates. We use a time step of 8 days which is the closest integer to the square root of 72. We compute 2 values, $VAR\left[x_{t+8}-x_t\right]$ and $VAR\left[x_{t+1}-x_t\right]$, and from (7);

$$(11) \qquad H_{S=8}=\frac{Log\left(\dfrac{VAR\left[x_{t+8}-x_t\right]}{VAR\left[x_{t+1}-x_t\right]}\right)}{2Log\left(8\right)}$$

The year-by-year Hurst measures are provided in Figure 3. The lowest value was H=0.137 (1991) and the highest was H=0.685 (1988). A trend analysis regressing H against year showed no statistical trend in the Hurst values (p= 0.61). We also computed a 'Hurst on Hurst' measure, using the same procedure as in (11) for the values in Figure 4 finding that $HoH=0.022$, which is highly ergodic. From a practical point of view this suggests, with high probability, that a low H will be followed by a higher H, and vice versa. This is a surprising result. In the absence of a theory, one would think that year-over-year weather patterns would be statistically independent of each other yielding a HoH value around 0.5. This does not appear to be the case. Thus, not only do our data indicate strong patterns of within-year fractionality, but that the patterns themselves are fractal. Exploring this further is beyond the scope of this paper.

[Insert Figure 4]

Figures 5 shows the scatter plot and power functions for cumulative rainfall (R^2= 0.1064) and damage intensity (R^2= 0.1154) with the Hurst coefficients. The damage intensity is taken from our measure of indemnity using a dynamic trigger as discussed in section 5, but for now it shows that a 1% increase in the Hurst coefficient corresponds on average with an increase in 'damage' of 0.717% (p=0.046). Similarly, a 1% increase in the Hurst coefficient corresponds on average with an increase cumulative rainfall by 0.3573% (p=0.056). Although the overall fit of these regressions is low, the relationships are interesting. Generally speaking, drought intensity increases with lower Hurst coefficients. The characteristic of low Hurst coefficient is that the seasonal rainfall patterns are mean reverting; in other words, the intertemporal covariance relationship is negative. Thus, in drought years an increase in rainfall is more likely in probability to be followed by a shortfall in rain. In contrast the high rainfall years with H>0.5 have a positive covariance suggesting that that increases in rainfall are reinforcing.

[Insert Figure 5]

Figure 6 provides the rainfall patterns within year, and these seldom match the expectation. The solid line below the dashed line captures rainfall deficits from the mean, and for 1998 (H=0.354), 2008 (H=0.326), and 2017 (H=0.607) early and late season droughts of consequence can be observed. In comparison, 1992 (H=0.567) follows the average path with a late season rainfall in excess of the average, while the rainfall in 1997 (H=0.633) exceeded the average over most time periods. The main point, of course, is that rainfall deficits can arise randomly throughout the season, causing crop damage not only within a phenological stage, but also across phenological stages. They also show short term path dependency in the various excursion patterns illustrated. These excursion paths, and patterns, rise above and fall below the cumulative mean rainfall. Our interest is in managing the risks when cumulative 21-day rainfall has an excursion below this 'barrier'. Our approach is discussed in the next section.

[Insert Figure 6]

b. Resolving Noah and Joseph effects with a dynamic trigger index insurance

In the previous sections we described the fractionality in within-season rainfall patterns. The range of Hurst coefficients give rise to observable and measurable excursions below the long run overlapping 21-day cumulative rainfall measures from 1983-2017. How to recognize and incorporate these excursions into WII model is discussed in this section. The background to this assessment is the results from a randomized control trial (RCT) to implement an insurance linked credit product called Risk-Contingent Credit (RCC) in 2017 to unbanked farmers in Machakos County in Kenya (for details of the study see Shee et al. 2015). The RCC product was in the form of a loan, which paid an indemnity if the accumulated rainfall between October 15th and January 15th (the long rains) fell below a rainfall trigger established in millimeters (mm) below the 15th percentile, or for an event that might occur about once in every 7 years. In our RCT design, 1150 sample households were randomly assigned to one of three research groups: treatment 1 (farmers assigned to receive traditional credit; 350 households), treatment 2 (farmers assigned to receive RCC; 350 households) or control (farmers assigned to receive no credit; 350 households).The simplicity of the design was intentional. The subject farmers had largely no interaction with formal banking services, let alone credit, and had no experience with weather (or any type of crop-related) insurance. However, in pre-experiment focus groups with farmers across the Machakos district, it was clear that failures of the rain (lack of rainfall or rainfall not occurring timely) were the biggest risks faced, and lenders also acknowledged that failure of the rain was the largest impediment to providing agricultural credit. Although we were aware of erratic weather patterns it was felt by the research team, local bank, and local insurer that a simple design for a first-time pilot of a new bundled credit product would be the least complicated approach, with product modifications and scaling up to follow.

Weather conditions in 2017, are depicted in the lower right panel of Figure 5, where a significant negative excursion developed, and the rains dropped below 20 mm between vegetative and flowering/maturity stages resulting in yield declines of more than 50% for many borrower farmers. However, because of high mid-season rainfall the insurance did not trigger. Fortunately, we had anticipated that possibility and used reserve funds to provide an indemnity equal to 50% of loan balances for those receiving RCC, but no indemnity for those receiving traditional loans.

For weather index insurance to be sustainable in the target area (and more generally Sub-Saharan Africa), an approach was needed that accounted for the fractional nature of rainfall discussed above. Although we find strong evidence of Noah- and Joseph-erratic phenomenon in our subject area, that does not imply that the weather risks are uninsurable. It does mean that in the traditional sense that simplified measures – including our own 2017-2018 cumulative rainfall model – will in many instances fail in the most basic efficiency measure of minimizing Type I (which is costly to the insurer) and Type II (which is costly to the farmer) error. Thus, there is a need to rethink insurance risks and indemnity structures. To confront the problems of fractional weather patterns as discussed above, we suggest here a different structure based on a 'dynamic trigger' that tracks current rainfall relative to historical norms, while taking into account whether the insured year is a 'high Hurst' year or 'low Hurst' year. This is discussed next.

To address the erratic nature of rainfall patterns with unpredictable excursions, and to reduce temporal basis risk we develop a 21-day event model. By 'event' we refer to any 21-day period in the insured season in which accumulated rainfall falls below 60% of the average accumulated rainfall for that district over the same historical 21-day period (on a calendar basis, ignoring leap years). Generally speaking, and as depicted in Figure 1, the pattern of rainfall is low at the beginning of the season, rising, and then decreasing towards the end of the season on January 15th. The triggering event is dynamic, in the sense that it maps onto the historical rainfall pattern, rising and falling accordingly.

The effects of a Dynamic Trigger are illustrated in Figure 7 for Central Machakos in 2015, with one small event, and 2017 with one small, and two significant events. The green line is cumulative rainfall, the blue line is the rolling 21-day cumulative rainfall, while the red dashed line is the Dynamic Trigger. Arrows indicate an 'Event', in which the rolling 21-day cumulative rainfall falls below the Dynamic Trigger. For example, the event horizon starts first at day 1- day 21, then day 2-day 22, day 3-day 23 until the end of the season day 73-day 93. In other words, there are 73 consecutive 21-day periods in the long rain period between October 15 and January 15th. Each period is examined to determine if the actual rainfall was below the trigger. If not, then the next sequential 21-day period is examined and so on. If the actual rainfall is below the corresponding trigger, an event is triggered. Subsequent events cannot be overlapping. For example, if no event is recorded at day 21 or day 22, but is recorded for day 23, another event cannot be recorded on day 24. The next possible date for a second event would be day 44, covering the 21 days between day 24 and day 44. Since events cannot be overlapping, at most 4 events could be recorded in a single long rain season.

[Insert Figure 7]

The rainfall deficit is measured by the difference between the rainfall trigger and actual rainfall in millimeters (mm). Monetizing this requires multiplying the deficit (in mm units) by a nominal unit (KSh/mm) – the tick value - to obtain an indemnity for that event in KSh. In our modelling we assumed that if an event was triggered, the minimal indemnity for the event was to be 500 KSh. The schema for the dynamic trigger RCC design is provided in the appendix.

Ultimately, the final premium is based on two variables, which we can vary. The first variable is the trigger level, and the second is tick value. Figure 8 illustrates the average premium rate and 2017/2018 indemnity based on a 10,000 KSh loan holding the dynamic trigger at 60% of average, and varying the tick value from 50 KSh/mm to 100 KSh/mm. At 50 KSh/mm the insurance yield is 15.78%. This increases to 25.74% for a tick of 100 KSh/mm.

[Insert Figure 8]

5. Concluding remarks

In assessing the hydrology of water flows in rivers, Mandelbrot and Wallis (1968) established certain properties of stochastic processes that were erratic. Although not often mentioned explicitly in the weather index insurance (WII) literature, there is a subjective reliance on the central limit theorem and an assumption that within-season weather patterns occur randomly, but with a convergent pattern. This is a weak assumption, and we urge for exploration into fractional properties of within-season variance and patterns of weather conditions (in our case precipitation) generally, and consideration of Joseph and Noah effects specifically. The erratic nature of weather patterns, and particularly participation, complicates the design of weather index insurance products. In this paper we investigated the erratic nature of rainfall patterns in Machakos County, Kenya and incorporated them in designing an operational insurance-linked credit product. The motivation for the paper was to ensure an optimal design of rainfall insurance that would minimize Type I and Type II error by reducing temporal basis risk.

Our findings point to an important warning sign for weather index insurance design. We find that the patterns of rainfall are indeed erratic and consistent with the Noah and Joseph descriptors discussed by Mandelbrot and Wallis (1968). The erratic nature of rainfall emerges from two statistical failings. The first is a breakdown of the convergence to a normal distribution around the mean of our 21-day rainfall measure. Instead we find that the distribution about the average is approximately lognormal, with an almost 50% higher chance of deficit rainfall below the mean versus adequate rainfall above the mean. Perhaps more important is our finding that the rainfall patterns obey Hurst's law. We find that the Hurst coefficients for the average pathway is about H=0.8, but the range of Hurst coefficients across all years ranged from a low below H=0.2 and a high above H=0.6. The average Hurst, however was not significantly different from 0.5, but this is meaningless in an insurance context.

Because of the erratic nature of rainfall, we develop a new approach to weather index insurance based upon the accumulated rainfall in any 21-day period falling below 60% of the long-term average for that same 21-day period. We argue that this approach is more satisfactory to matching drought conditions within and between various phenological stages of crop growth. While this new approach reduces Type I and Type II error, it comes at higher cost.

Characterizing weather patterns according to their fractional properties is no easy task and should be with longer time-series data. In our case, we only had data from 1983 to 2017 which may be a limitation in this study, however, rainfall data prior to 1983 are hardly available. In addition, our within-year measures are also limited in time, but this is unavoidable given the calendar dates of the long-rain growing season investigated, and the time-discontinuities between growing seasons. Nonetheless, the central ideas in this paper can be expanded to other crops and regions and be considered in future developments of weather index insurance for agriculture.

At the macro level the ideas in this paper can be extended to the understanding of poverty traps. Barrett and Swallow (2006) provide an informal approach to what they refer ‘fractal poverty traps’ measured by scaling across farm types and industry actors, and space. Their bifurcated model explains the coexistence of high and low (multiple) equilibrium levels of productivity and income and high and low rates of economic growth. The source of their fractal poverty trap are exogenous shocks and market failure, including the failure of insurance and credit markets to develop. From our results we can observe how both short and long run fractional excursion patterns (the Joseph effect) can break through the threshold barrier of the dynamic trigger. With multiple equilibria derived from degrees of resilience, our approach to balancing business and financial risks for small holder farmers using Risk-Contingent Credit would, in theory at least, provide a source of resilience that could ultimately reduce the number of low-level equilibria in an agricultural economy. How fractional weather patterns can give rise to fractional poverty traps is worthy of further study.

Finally, we do not believe at this time that the Hurst coefficient can be used directly in an indemnity formula for WII. A high Hurst, as discussed in the text, can lead to both negative and positive excursion patterns, and these would have to be parsed out into a conditional probability framework. Nonetheless, we believe that that viewing weather patterns and WII through this fractional lens provides at least a first-step in placing within-season WII for agriculture on a more solid scientific footing.

Appendix

Model schema for the design of weather index insurance for a risk-contingent credit product

$$
\begin{aligned}
&Z_s = Dynamic\,Trigger\ for\ any\ 21-day\ period\,(mm) \\
&s = Rolling\ 21-day\ counter \\
&R_s = Actual\ Long\ Rain\,Total\ for\ any\ 21-day\ period\,(mm) \\
&R_s = \sum_{t=1}^{21} R_t \rightarrow R_1 = \sum_{t=1}^{21} R_t, R_2 = \sum_{t=2}^{22} R_t, \ldots R_{72} = \sum_{t=72}^{93} R_t \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\psi = Tick\,Value\,(Ksh\,/\,mm) \\
&If\ R_s < Z_s \\
&Indemnity_s = \left(Z_s - R_s\right) \times \psi \\
&Payout_s\,(Ksh) = Max\left(500, \left(Z_s - R_s\right) \times \psi\right) \\
&Total\ Indemnity = \sum_{k=1}^{Max=4} Max\left(500, \left(Z_{s\rightarrow k} - R_{s\rightarrow k}\right) \times \psi\right) \\
&Insurance\,Premium\,Rate = \theta \\
&\theta = \frac{E\left[\sum_{k=1}^{Max=4} Max\left(500, \left(Z_{s\rightarrow k} - R_{s\rightarrow k}\right) \times \psi\right)\right]}{10{,}000} \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&f = Loan\,Principal\,(Ksh) = Loan\,Request + Insurance\,Premium = Loan\,Request \times \left(1+\theta\right) \\
&r = effective\,annual\,interest\,rate \\
&T = time\,to\,loan\,repayment \\
&Farmer\ Repayment = f\left(1+\frac{r}{T}\right) - \sum_{k=1}^{Max=4} Max\left(500, \left(Z_{s\rightarrow k} - R_{s\rightarrow k}\right) \times \psi\right)
\end{aligned}
$$

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Figures:

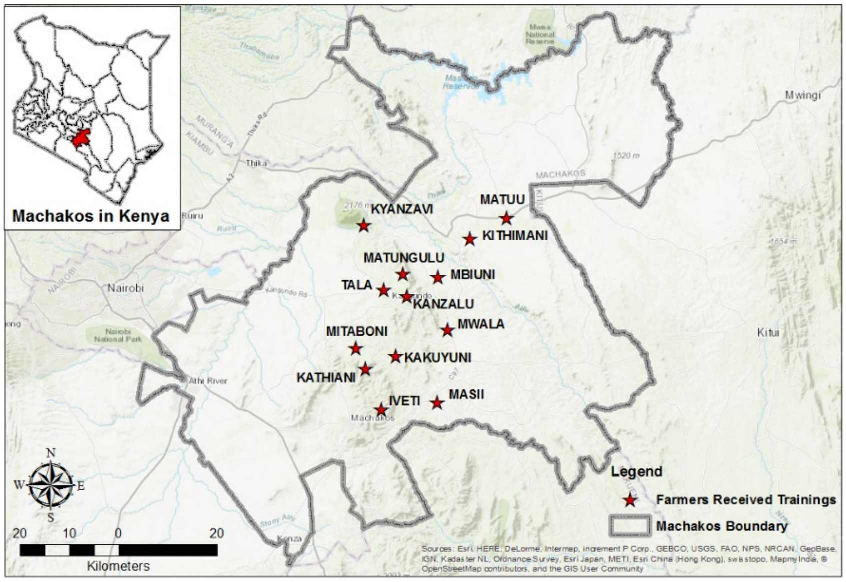


Figure 1 Map of study area- Machakos county, Kenya

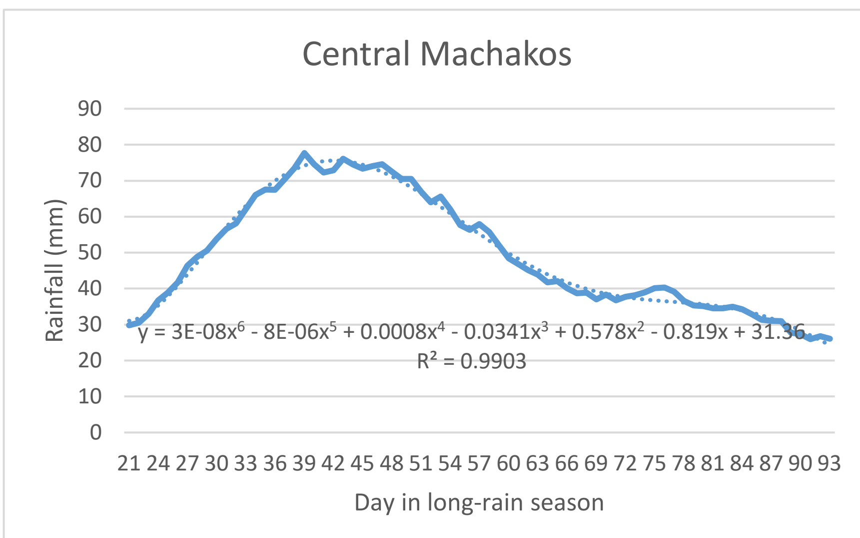


Figure 2:21-Day moving average cumulative rainfall for Machakos county based on CHIRPS data; average of daily rainfall of long rain (15 October-15 January) from 15 October 1983 to 15 January 2018, with fitted 6th order polynomial smoothing

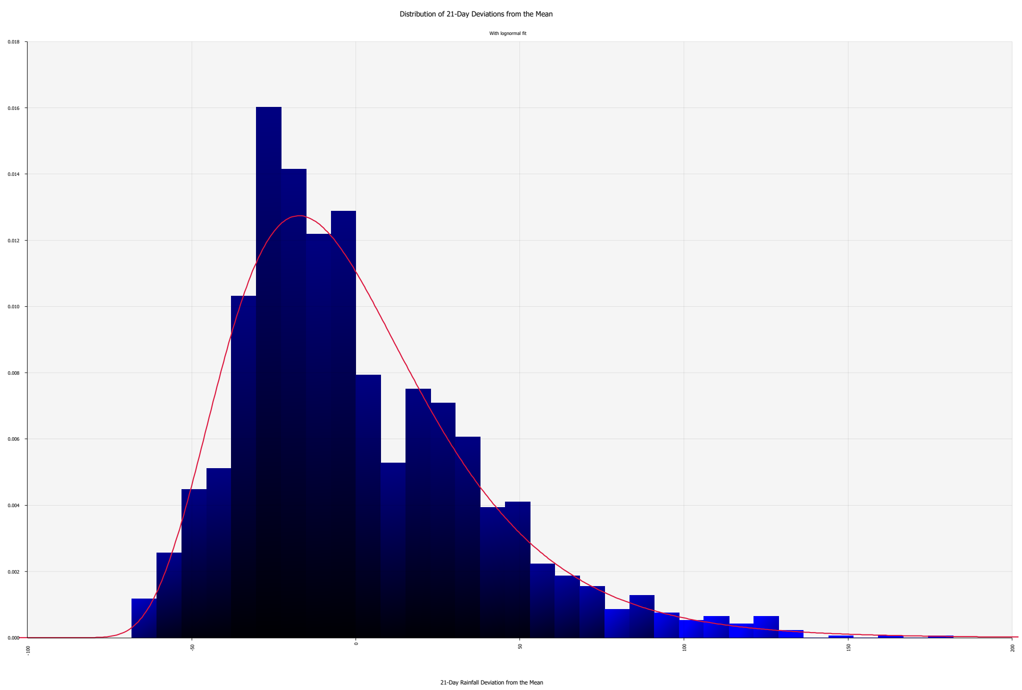


Figure 3: Distribution of 21-Day deviations from the mean

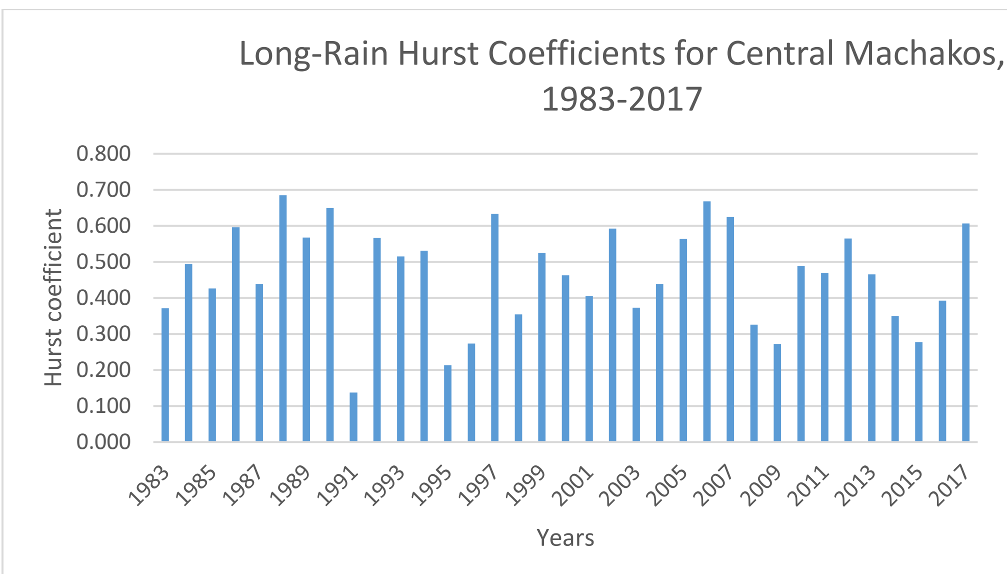


Figure 4: Hurst coefficients for long rains in Central Machakos

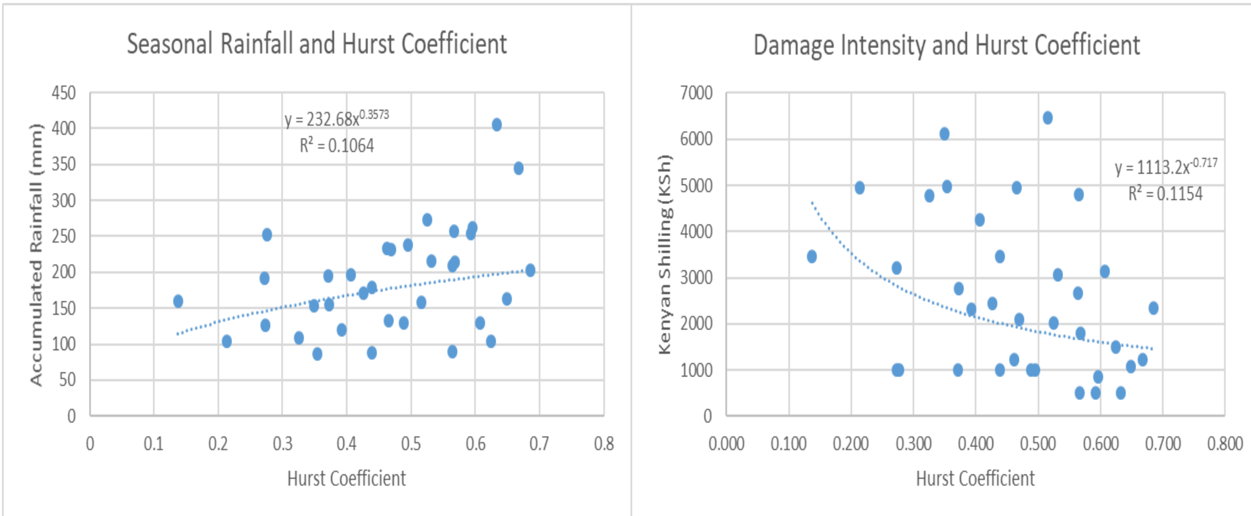


Figure 5: Seasonal rainfall and insurance indemnity with Hurst coefficients

Figure 6: Cumulative 21-Day rainfall measures compared to average 21-day measure, 1983-2017. Solid line represents moving 21-day rainfall and dashed line represents dynamic trigger. Years 1983 (H=0.371) and 1992 (H=0.567) were years of adequate rainfall. The year 1997(H=0.633) had the highest recorded seasonal rainfall (405 mm), 1998 (H=0.354) had the lowest seasonal rainfall (86.9 mm). The years 1998 (H=0.354), 2008 (H=0.326), 2017 (H=0.607) as illustrated were amongst the worst drought years in 1983-2017 in the Machakos county.

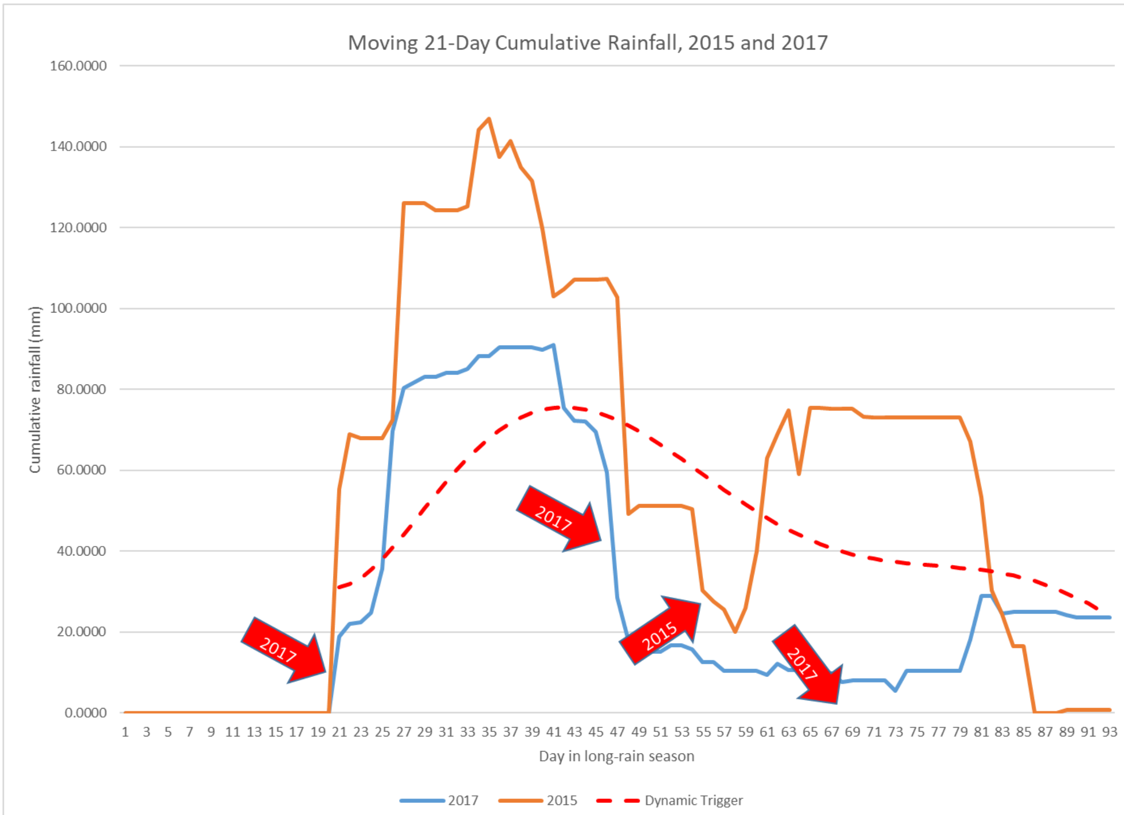
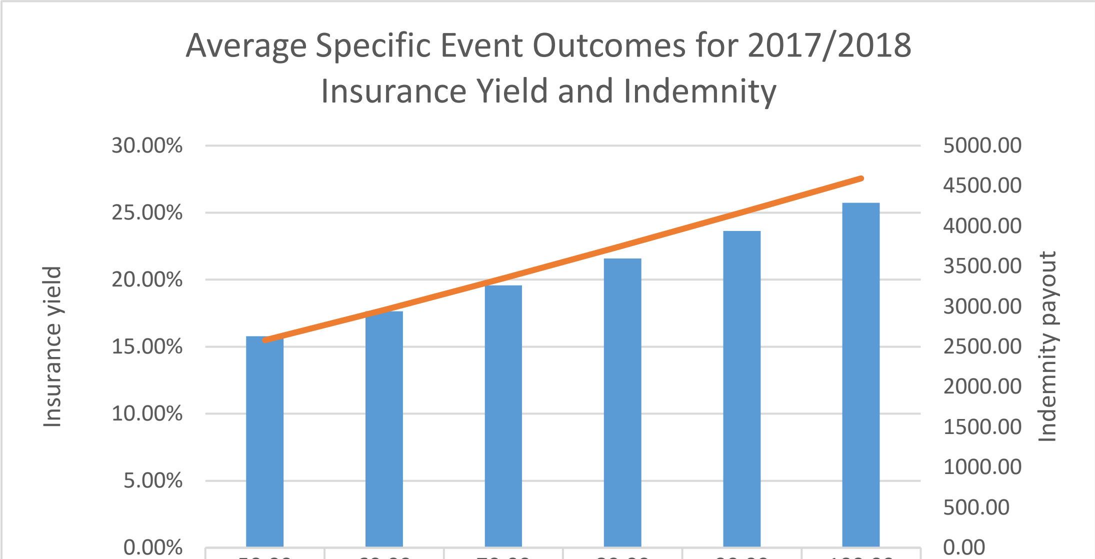


Figure 7: Dynamic trigger rainfall insurance for long rain in Central Machakos in 2015 and 2017. Blue line represents moving 21-day rainfall for 2017, Orange line shows moving 21-day rainfall for 2015, and arrows represent the days when an event is triggered)



	50.00	60.00	70.00	80.00	90.00	100.00
yield	15.78%	17.64%	19.58%	21.59%	23.64%	25.74%
Payment	2581.00	2957.00	3351.00	3755.00	4171.00	4593.00

Figure 8: Average insurance yield and indemnity payment for dynamic trigger specific event insurance