Noah and Joseph Effects in the Application of 'Dynamic Trigger' Rainfall Risk-Contingent Credit in Sub-Saharan Africa

By

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Introduction

In Mandelbrot and Wallis's classical 1968 exploration into the fractional properties of water flow they engagingly introduce the problem with biblical references to the 'Noah" effect, which refers to the fact that extreme precipitation can be very extreme, and the 'Joseph Effect' in which a period of high or low precipitation can be extremely long[1]. The extremities of rainfall are bounded from below at zero and are theoretically limitless from above, although not unlikely without bound. To what extent either the Noah Effect or the Joseph Effect impacts the human or natural condition is not immutable and is determine prima facie by the mathematical mix of frequency, duration and intensity and with respect of the ecology under consideration. In the ecology of rainfall and agriculture that ecology is not static. What might be an abundance of precipitation in one stage of a plant's phenology, my be insufficient in another stage, and inadequate in yet another.

In the context of agricultural weather insurance – the broader subject matter of this paper – these ideological considerations have taken on manifest importance, although by admission, not exactly the storyline originally intended. To explain: the economic problem we seek to address is the design of a specific-event weather insurance product that can be imbedded into a credit product to provide relief to farm borrowers in time of drought, reduce risk-rationing and increase demand for credit, and provide at least a partial substitute for collateral and reduce financial risk and exposure to lenders, who are reluctant to lend to agriculture because of the very weather risks we seek to insure. To address the phenological problem we develop in what we refer to as a dynamic trigger. This trigger establishes an indemnity if the accumulated rainfall in any 21-day period is below 60% of the historical average rainfall in that same 21-day period for a given year. More on this later, but when we examined the 'average' path of overlapping 21-day measures and took the deviation of each year's equivalent measure we found the distribution of the difference to be non-normal! Indeed we find it to be close to a lognormal distribution with the probability that below-normal rainfall in our study region was approximately 50% more likely than above normal rainfall. The failure of normality suggest also a failure in the Gauss-Markov assumption normally assumed in a first-guess approach to statistical assumption. But this also comes with a possible failure in the independence assumption and Brownian

[1] Mandelbrot, B. and J.R. Wallis (1968)."Noah, Joseph and Operational Hydrology" Water Resources Research. 4(5):909-918

presumption of the historical time-path of our data series – as limited as it is. As Mandelbrot and Wallis (1968) point out the failure to recognize the non-Markov possibilities would greatly underestimate the duration and intensity of the longest drought. Exploring further, we deployed the within-year variance-ratio measure of the Hurst coefficient and found that a) when taken as an average (1983-2017) the within-year Hurst coefficient is approximately H=0.8, b) the within-year Hurst coefficient varies widely from a low of 0.137, a high of 0.687; and c) an average H of 0.466, which with a standard deviation of 0.1369, is not statistically different from H=0.5. Clearly, the Hurst of the average is nowhere near that average Hurst. Combined, these observations should encourage researchers in the field of developing or evaluating the many varieties of weather index insurance models to treat with greater seriousness the combined Noah and Joseph effects. Perhaps this is stating the obvious. However, recognizing that the Noah Effect is essentially dealing with a volume metric of extremes with reference to some measure not considered extreme, it is considered independently of how precipitation is patterned. A weather 'pattern' is distinctively non Markovian in the sense that over some measure of scale there is measureable correlation between any time date t, and some other date t-s or t+s. On patterns, the Joseph effect is more important since these correlated patterns can be impactful in the small (within years) or in the large (between and across years). Of course weather patterns come and go, and are mixed in terms of frequency, duration, and intensity, but to assume a priori that H=0.5 is a false-Markov understanding of weather risks and suggests a good chance that the insurance design will fail to accurately reflect the extremities of indemnity.

Scaling Properties and Erratic Weather Processes

Mandelbrot and Wallis (1968) point to three characterizations of stochastic processes that might give rise to a Brownian Gauss-Markov process. The first is that some process X(t) will satisfy the law of large numbers in the sense that its expected value tends to a limit, E[X(T)] as T approaches infinite. The second is with respect of the central limit theorem in that for large T the distribution around the average becomes approximately Gaussian for T to infinite. The third deals with the scaling properties of the process and independent increments such that for large T, X(t+s)-X(t) will be statistically independent from another measure, say X(t)-X(t-s) for any arbitrary s, or perhaps more generally VAR(X(t+Ks)-X(t))/VAR(X(t+s)-X(t)) = K. This is equivalent to the linear-in-variance property of standard Brownian motion.

When one or more of these conditions are violated, Mandelbrot and Wallis (1968) refer to the processes as being 'erratic'. Thus 'Joseph-erratic' might refer to a phenomenon of an extraordinary term of wetness or dryness within a time span such that localized path dependence and measurable correlations are not obscured or mitigated by the law of large numbers. 'Noah-erratic' behavior occurs when the intensity of the weather event (precipitation or lack thereof) is so great as to affect the average of the measured event for many periods (e.g. years) after the event occur. Notably, both Joseph- and Noah-erratic behavior can occur simultaneously and feed off each other.

To get a sense of the scaling properties that might give rise to erratic behavior consider the ordered set of measures (e.g. precipitation) $X_T = \{x_{t+S}, x_{t+S-1}, x_{t+S-2} ... x_t\}$. For convenience of illustration set $t = 0$ and $S = 100$. Then, the difference $x_{100} - x_0$ can be expressed equivalently as

$$(1)\quad x_{100} - x_0 = x_{100} + (x_{99} - x_{99}) + (x_{98} - x_{98}) ... + (x_2 - x_2) + (x_1 - x_1) - x_0$$

or

$$(2)\quad x_{100} - x_0 = (x_{100} - x_{99}) + (x_{99} - x_{98}) ... + (x_2 - x_1) + (x_1 - x_0)$$

Assuming equivalence of any $(x_t - x_{t-1})$

$$(3)\quad \begin{aligned} E[x_{100} - x_0] &= E[(x_{100} - x_{99}) + (x_{99} - x_{98}) ... + (x_2 - x_1) + (x_1 - x_0)] \\ &= E(x_{100} - x_{99}) + E(x_{99} - x_{98}) ... + E(x_2 - x_1) + E(x_1 - x_0), \\ &= 100E(x_1 - x_0) \end{aligned}$$

Or more generally

$$(4)\quad E[x_{t+S} - x_t] = SE(x_1 - x_0).$$

The variance of the differences can be expressed as

$$(5)\quad VAR[x_{t+S} - x_t] = \sum_{t=1}^{S} E[(x_t - x_{t-1}) - E(x_t - x_{t-1})]^2 + COV[x_{t+S} - x_t],$$

where

$$(6)\ \ COV\left[x_{t+S}-x_t\right]=2\sum_{(i\in t)=1}^{S}\sum_{(j\in t)=1}^{S}E\left[\left(x_i-x_{i-1}\right)-E\left(x_i-x_{i-1}\right)\right]\left[\left(x_j-x_{j-1}\right)-E\left(x_j-x_{j-1}\right)\right], i\neq j.$$

It is from this measure of variance that the scaling properties related to erratic behavior arise. The scaling can be defined by S^{2H}, where H is the Hurst coefficient. The Hurst coefficient plays a crucial role in the identification of fractional properties of time series. We can restate variance in terms of the Hurst scaling rule as,

$$(7)\ \ VAR\left[x_{t+S}-x_t\right]=VAR\left[x_t-x_{t-1}\right]S^{2H}.$$

Moreover, since there is no reason to believe a priori that $VAR\left[x_t-x_{t-1}\right]\neq VAR\left[x_{t+1}-x_t\right]$ we can restate the scaled variance measure as

$$(8)\ \ VAR\left[x_t-x_{t-1}\right]S^{2H}=VAR\left[x_t-x_{t-1}\right]S+COV\left[x_{t+S}-x_t\right],$$

and from this restate covariance as,

$$(9)\ \ COV\left[x_{t+S}-x_t\right]=VAR\left[x_t-x_{t-1}\right]\left(S^{2H}-S\right).$$

Expressed in this way the covariance term defines the Brownian set and the nature of dependence in fractional processes. The relationships are as follows;

$$(10)\qquad S\Leftarrow\left\{\begin{array}{l} H=\frac{1}{2}\rightarrow COV\left[x_{t+S}-x_t\right]=0,\ S=\frac{VAR\left[x_{t+S}-x_t\right]}{VAR\left[x_t-x_{t-1}\right]} \\ H>\frac{1}{2}\rightarrow COV\left[x_{t+S}-x_t\right]>0,\ S>\frac{VAR\left[x_{t+S}-x_t\right]}{VAR\left[x_t-x_{t-1}\right]} \\ H<\frac{1}{2}\rightarrow COV\left[x_{t+S}-x_t\right]<0,\ S<\frac{VAR\left[x_{t+S}-x_t\right]}{VAR\left[x_t-x_{t-1}\right]} \end{array}\right\}.$$

First, for $H=\frac{1}{2}$, the covariance term collapses to zero. In this state there is no intertemporal correlation between changes in the measure. This defines a pure Gauss-Markov, memoryless process of a standard Brownian motion and the linear-in-variance assumption. Here, the variance of the measure over a hundred days, or months, or years is 100 times the 1-step measure in days or months or years. And it is predictable. This condition will satisfy all three of the characteristics identified by Mandelbrot and Wallis (1968) and is consistent with the usual interpretation of a random walk. However for $H < \frac{1}{2}$ and $H > \frac{1}{2}$ the scaling properties are not memoryless, and thus violate the Markov property. For $H > \frac{1}{2}$, systemic positive correlation compounds the variance so that the variance of the measure over 100 days, or months, or years will be greater than 100 times the 1-step measure. It is persistent. Likewise, for $H < \frac{1}{2}$, the covariance will be systemically negative so that an increase in the measure of some time scale will, in probability, reverse itself in a mean-reverting or ergodic way. The variance of the measure over 100 days, or months, or years, will be less than 100 times the variance of the 1-step measure.

When $H \neq \frac{1}{2}$ the third condition is violated and this in return gives rise to Noah- and Joseph-erratic behavior. Whether variance is expanding in scale or contracting in scale, the process becomes far less predictable. When the process is erratic-persistent the precipitation patterns are subject to longer excursions in wetness and dryness; when the process is ergodic-erratic the patterns of rainfall are more oscillatory with increasingly shorter excursion paths as H gets smaller.

It becomes evident then that Hurst's law has something important to add to the broader discussion of weather index insurance. We will show presently for the case of Machakos County, Kenya in Sub-Saharan Africa that indeed Hurt's law holds and in doing so violates Mandelbrot and Wallis's (1968) third condition; but equally important is our finding that the second condition- that of Gaussian-normal error around the mean path of our rainfall measure - also fails! Before revealing these results, we next discuss the significance of weather patterns for agricultural productivity.

Risk and Agricultural Productivity

Turvey (2001) notes that most perils insured in crop production can be linked to specific event weather events affecting via temperature and rainfall interactions evapotranspiration, phonological growth, and the rise of pestilent and viral infestations[2]. Norton et al (2016) for example suggest that weather insurance ought not to be viewed as a perfect substitute for multiple peril crop insurance, but as a risk-transfer instrument should be specifically targeted towards covariate events that are weather-sensitive such as Karnal Bunt and Stewart's disease. Particular weather conditions can also give rise to insect infestations for which Richards et al (2006) suggest 'bug options'[3].

The weakness in the application of weather insurance in both developing and developed agricultural economies is the existence of 'basis' risk. Generally speaking, basis risk, refers to the observed variability in an insured asset that is seemingly uncorrelated, or weakly correlated, with a proxy measure of risk. Basis risk can arise spatially by distance, altitude and geomorphology (Norton et al (2012), Heimfarth et al (2006), Heimfarth and Musshoff (2011), Woodard and Garcia 2008)[4]. However it is increasingly being recognized that basis risk also arises with the patterning of weather within a season at a specific location. Intra-seasonal basis risk deals with the failure of a specified index to adequately capture within-season variability, leading to excessive type I or type II errors. Type I error refers to insurance payments when no crop damage is observed, while type II error refers to situations in which crop damage is observed but no indemnity is paid. In a more general way we can say that added specificity to the weather index ought to reduce both Type I and Type II error, at least in probabilistic terms. One approach to doing this is to recognize that rather than a single index for an entire season (e.g. based on cumulative rainfall), the growing season can be broken up into multiple events.

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[2] Turvey, C.G. (2001). Weather Derivatives for Specific Event Risks in Agriculture. Review of Agricultural Economics. 23(2):333-351

[3] Turvey, C.G. and M. Norton (2008) "An Internet Tool for Weather Risk Management." Agricultural and Resource Economics Review. April:63-78

Richards, Timothy J. , James Eaves, Valerie Fournier, S.E. Naranjo, C.-C. Chu, T.J. Henneberry, (2006) "Managing economic risk caused by insects: bug options", Agricultural Finance Review, Vol. 66 Issue: 1, pp.27-45,

[4] Norton, M.T., C. Turvey and D. Osgood (2012) "Quantifying Spatial Basis Risk for Weather Index Insurance". J. risk Finance 14:20-34

Woodard, J.D. and P. Garcia (2008) "Basis risk and Weather Hedging Effectiveness" Agricultural Finance Review 68:99-117

Heimfarth, Leif Erec, Robert Finger, Oliver Musshoff, (2012) "Hedging weather risk on aggregated and individual farm-level: Pitfalls of aggregation biases on the evaluation of weather index-based insurance", Agricultural Finance Review, Vol. 72 Issue: 3, pp.471-487,

Heimfarth, Leif Erec, Oliver Musshoff, (2011) "Weather index-based insurances for farmers in the North China Plain: An analysis of risk reduction potential and basis risk", Agricultural Finance Review, Vol. 71 Issue: 2, pp.218-239, An obvious starting point for multiple events is to examine the phenology of crop production. In general there are three stages of crop growth; the vegetative stage from germination to panicle initiation, the reproduction phase from panicle formation to flowering, and the ripening phase from flowering to the final formation of grain. Conradt, Ginger and Sporri (2015), Dalhaus and Finger (2016) , Dalhaus, Musshoff and Finger (2018), and Shi and Jiang (2016) have all explored models for including phenology in an index for weather insurance[5]. Shi and Jiang (2016) created a composite index based on sub-period weather data that covers the three stages of growth. Using rice in China, the vegetative stage covered seedling, tillering and stem elongation (31 days + 20 days + 20 days = 71 days), the reproductive phase included panicle formation through flowering (33 days + 8 days = 41 days), and the ripening phase that covered an additional 57 days. All told, the insurable season covered 169 days. Using a two-stage procedure they mapped discrete-time days-in-season for each phase, and continuous time observations on rainfall, relative humidity, sunshine and temperature to construct a parametric composite index.

Dalhause and Finger (2016) broke the growing season into multiple parts and using historical and observational data on crop phenology for German wheat designed multiple event index insurance for which the farmer could choose amongst its 'calendar' parts. They find that use of phenological observations significantly reduced basis risk. Likewise, Conradt et al (2015) investigated a flexible index insurance plan for Growing Degree Days in Khazakstan. "Flexibility' in theor context was to determine the beginning and end periods of the phenological growth stages (start and end dates) which vary fro year to year. The advantage to weather risk management is the recognition that within season weather patterns are not constant from year to year, and will generally have different start and end dates from one year to the next. Depending upon the start-end date signals the days over which insurance is to be calculated. For example across years the start of the growth phase could begin at any time between June 2 and June 14th,

[5] Conradt, S., R. Finger, and M. Sporri (2015) "Flexible Weather Index-Based Insurance Design" Climate Risk Management 10:106-117
Dalhaus, T. and R. Finger (2016) "Can Gridded Precipitation Data and Phenological Observatios Reduce Basis Risk of Weather Index-Based Insurance?" Weather, Climate and Society 8 (Oct):409-419
Dalhaus, T., O. Musshoff and R. finger (2018) "Phenology Information Contributes to Reduced Temporal Basis Risk in Agricultural Weather Index Insurance" Scientific reports. 8:46
Shi, Hong and Zhihui Jiang (2016) "The Efficiency of Composite Weather Index Insurance in Hedging Rice Yield: Evidence from China" Agricultural Economics 47:319-328

while the end date had much greater variance between July 12th and August 12th with a mid date of July 24th. When computing a weather index for insurance a warm year might have the start date 10 days earlier than in a cold year. In terms of phenological growth, seeds will germinate much sooner in warm years than cold years, but if the dates for index measurement were fixed the index might weigh more heavily the effects of a cool year versus a warm year. A similar approach was deployed by Dalhaus et al (2018) for German wheat, finding that developing weather index insurance using published phenological observations increased farmers' utility and reduced financial exposure to drought risk. The approach was to establish begin and end dates of each growth stage using growing degree days ,and then accumulating rainfall within each stage to develop and indemnity structure.

These studies raise a certain number of issues for weather index insurance. Most critical is when the growing season starts. This is random. In our Kenya example the reference date for the beginning oflong rains is October 15th. But farmers have told us that some seed early in case the rains come early, while others withhold seeding until the rain has observably arrived. Both are rationally precautionary. However, if farmers spread seeding across a two week period defining risks according to specific calendar dates is a wobbly venture. The targeting of specific events as prescribed in Turvey (2001) would be effective only if planting and weather conditions were relatively homogenous across farms in a particular heat unit isocline. But variance in weather conditions – heat and rainfall – can shift the stages across calendar date boundaries, rendering the ideal of specificity benign. The cure would be to widen the date range so that in probability the shifting patterns of weather affecting a particular stage of growth, for example silking in corn, would adequately be captured. From this point of view, basing weather index insurance on broader phenological stages is a sensible approach to balancing Type I and Type II error.

Even so, it is troubling that the patterning of weather variables should be confined to proximal calendar dates of the various growth stages as if each stage can be treated as an individual and independent peril. While clearly adequate for reducing within-season basis risk, the approach does not take into account overlapping perils. For example if a late vegetative stage drought overlaps or spans the next stage tillering it is entirely possible that neither event would trigger a payout yet crop damage would be measureable. Turvey (2001) and also Turvey and

Norton (2008) address this problem in their specific event approach. From Turvey's (2001) Ontario study an insurance that would payout if rainfall in any non-overlapping 14 day period (an event) was equal to zero between June 1 and July 31 with up to 4 events reported for Ottawa a 91.4% chance of no events, 7.6% chance of 1 event, and 1% chance of 2 events. For weather stations in Welland and Woodstock, Ontario, the percentages were 81.6%, 16.5%, and 1.9%; and 84.8%, 12.4% and 2.9%. The frequencies measured over the same time span are indicative of the spatial basis risk (see Woodard and Garcia (2008) and Norton et al (2014?)) discussed in the literature, but also the patterning of rainfall within the season. From Turvey and Norton (2008, Figure 5A) a rainfall event with 21-days having rainfall below 1" at Ithaca NY showed a 39.19% chance of zero events, 31.08% chance of 1 event, a 20.27% chance of two events and a 9.46% chance of three events. The conditional probabilities were that if one event was observed there was a 48.9% chance of a second event, and 15.56% chance of a third event; and if second event was observed there was a 31.83% chance of a third event.

The advantage of a specific event approach to measuring weather risk is that by slicing the growing season into fixed day events that are overlapping in measurement, but not overlapping on indemnity, is the added flexibility to capture risks within each phenological stage, but also across the temporal boundaries of the phenological stages. This would further reduce the potential for Type II error, as well as Type I error. However, the specifications discussed in Turvey (2001) and Turvey and Norton (2008) are also imperfect in the sense that the nature of the event is assumed to be constant for all all events, i.e. rainfall less than 1" in 21 days. In reality the seasonal patterning of rainfall should not be assumed constant, nor should the triggering event. Indeed, cropping systems evolve according to the historical weather patterns that define the local ecology and growing conditions. It is ludicrous, for example, to treat equally Ardmore Oklahoma – the center point of the 1930's dust bowl – with Ithaca NY to its northeast. From Turvey and Norton (2008) Ardmore averaged 9.08" of rainfall between June 1 and August 31, while Ithaca averaged 10.74". A specific event metric that would pay out if rainfall fell below 5" over this period would have paid out nearly once in every 5 years (21.21%) in Ardmore, but would never pay payout in Ithaca; Likewise a specific event defined by 7 straight days with daily temperatures exceeding 90F would pay out once in only 3 of 100 years in Ithaca, but would pay out on 4 or more distinct non-overlapping events in nearly 93 of every 100 years (92.71%) in Ardmore. While these exemplify the spatial differences in the large, Norton et al (2012) illustrate how basis risk correlates with differences in distance, altitude, and direction in longitude and latitude from a given weather station.

Fractional Patterning of Rainfall in Sub-Saharan Africa

But what about the patterning of rainfall within a season? In the subject area at Machakos County, Kenya where we are implementing a randomized control trial to investigate bundled, or risk-contingent, credit based on long and short rains it is generally agreed (as previously discussed) that the long rains start at October 15th and end on January 15th. Typically, rainfall is not evenly distributed but starts low, rises to a mid-season peak and then diminishes thereafter. In fact the pattern appears to be uniformly described by a 6th order polynomial across all of the districts we examined. A typical pattern is illustrated in Figure 1, for Central Machakos, for the average of the 21-day cumulative rainfall from 1983 to 2017.

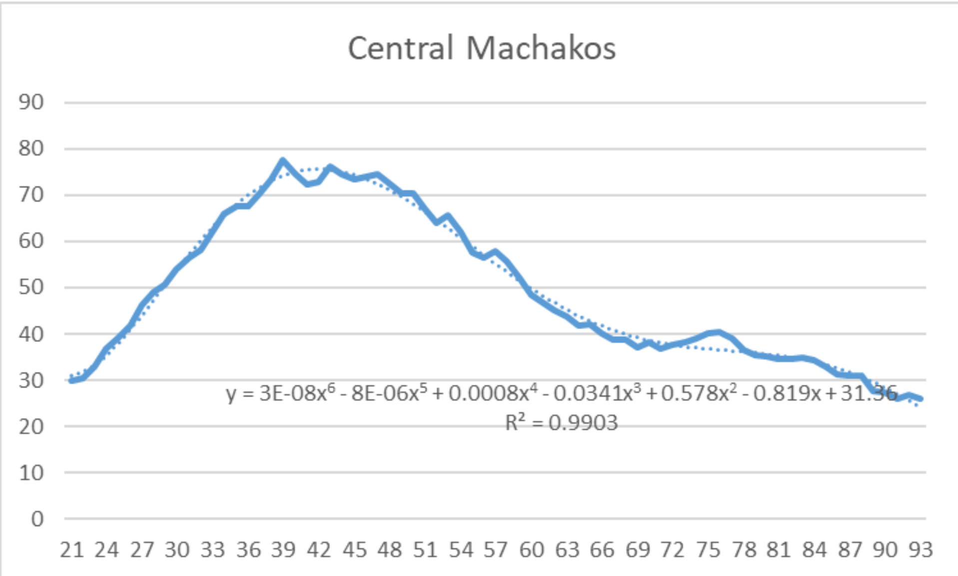


Figure 1:21-Day moving average cumulative rainfall for Machakos county based on CHIRPS data; average of 1983-2017, with fitted 6th order polynomial smoothing.

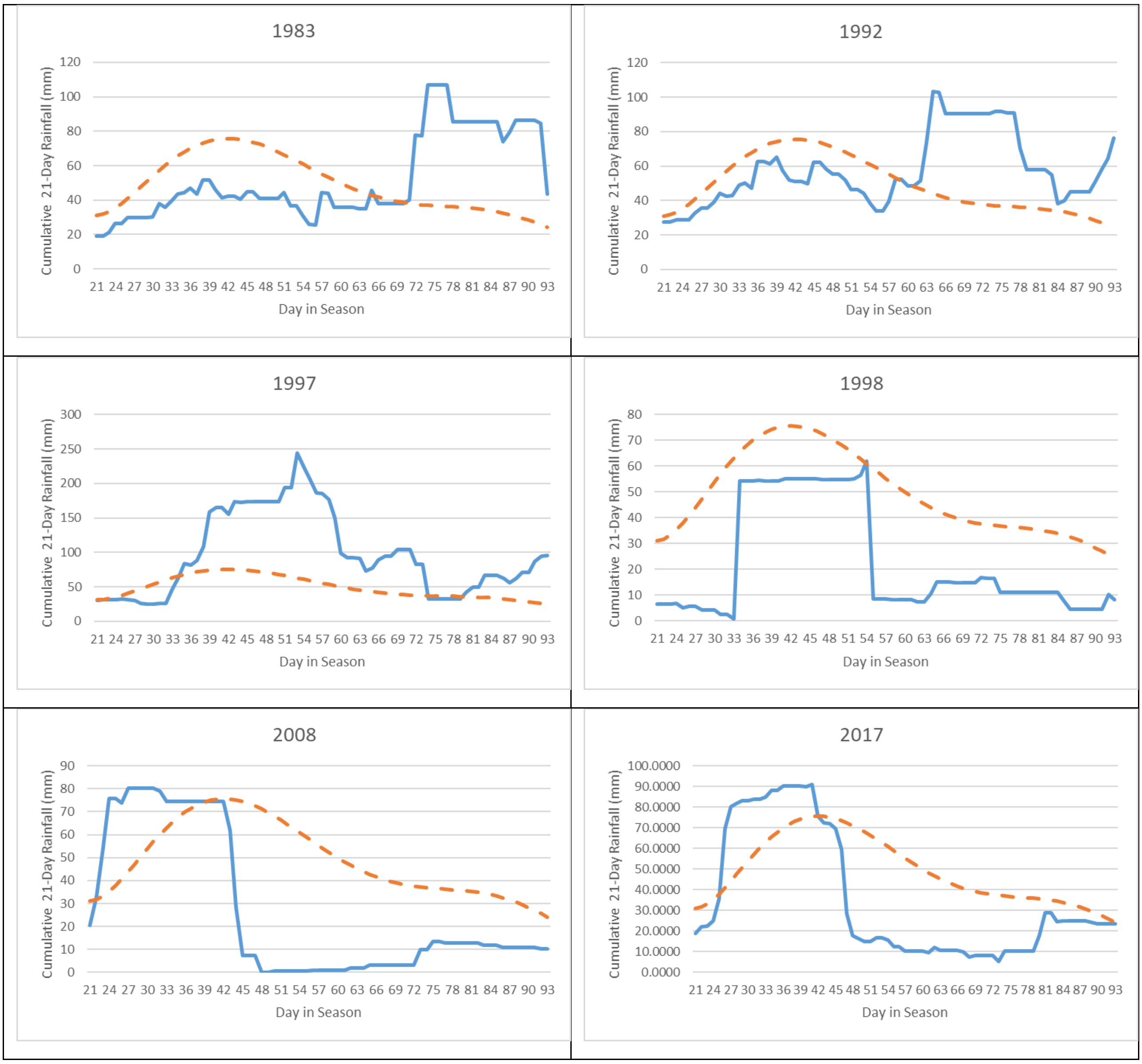


Figure 2:Cumulative 21-Day rainfall measures compared to average 21-day measure, 1983-2017. The year 1997 had the highest recorded seasonal rainfall (405 mm), 1998 had the lowest seasonal rainfall (86.9 mm). The years 1998, 2008, 2017 as illustrated were amongst the worst drought years in the Machakos region.

The issue of temporal basis risk is vividly portrayed in Figures 1 and 2. Figure 1 is the average of 21-day cumulative rainfall measures but this does not tell the whole story. Figure 2 provides the rainfall patterns within year, and these seldom match the expectation. The solid line below the dashed line captures rainfall deficits from the mean, and for 1998, 2008, and 2017 early and late season droughts of consequence can be observed. In comparison, 1992 follows the average path with a late season rainfall in excess of the average, while the rainfall in 1997 exceeded the average over most time periods. The main point, of course, is that rainfall deficits can arise randomly throughout the season, causing crop damage not only within a phenological stage, but across phenological stages. They also show a path dependency in the sense that a deficit in one period would likely be predictive of a deficit in the next or subsequent periods.

Fractional Weather Patterns

Figure 3 illustrates the day-to-day probability distribution of the deviation of the recorded weather pattern from the mean (N=2478). A typical assumption of randomness is that these deviations be normally distributed, however we find that the distribution is more closely aligned with the log normal distribution. The mean of the distribution is zero, as expected, but the skew is 1.0123 with kurtosis of 4.178. The modal value of -24.86 is negative, and the probability that a deviation is negative is 59.8% against 40.2% chance of a positive deviation. In other words, if 21-day cumulative rainfall is to deviate from the long run mean, it is 50% more likely to be a negative deviation than a positive deviation.

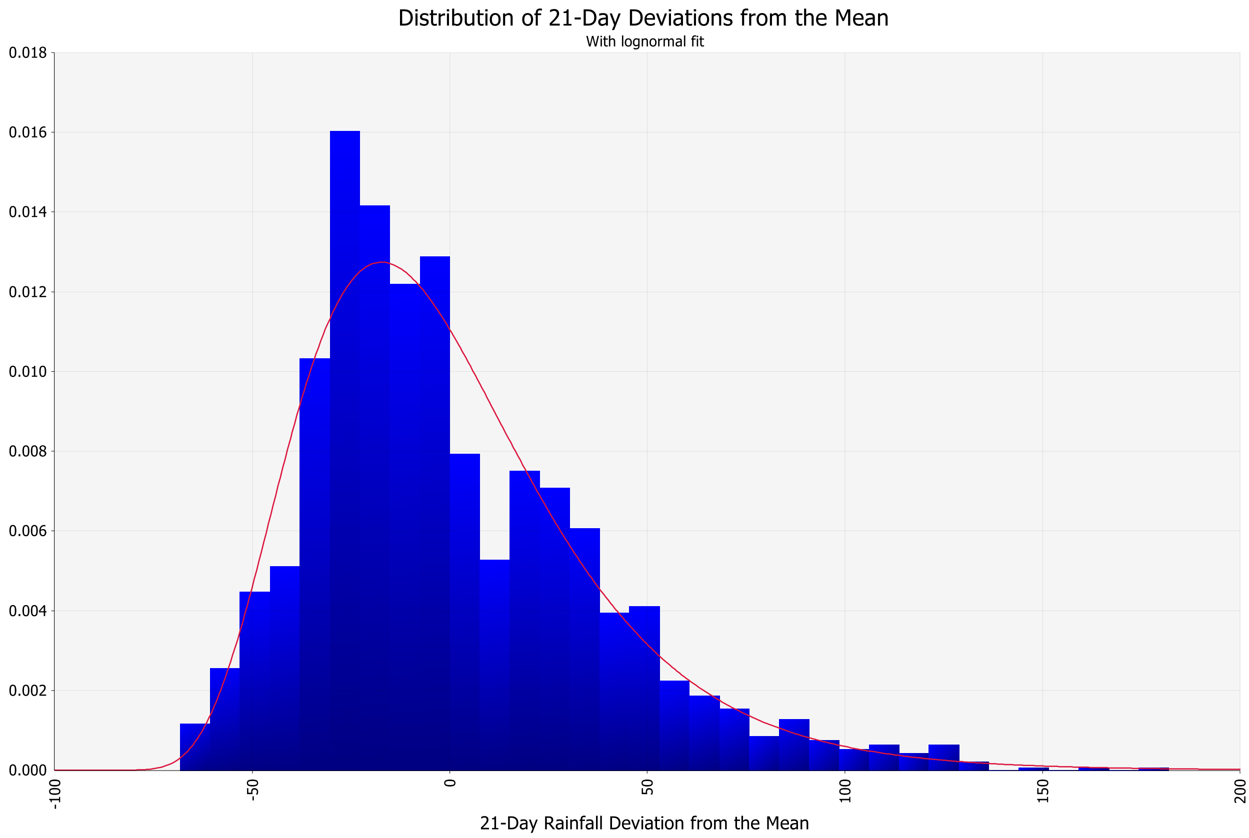
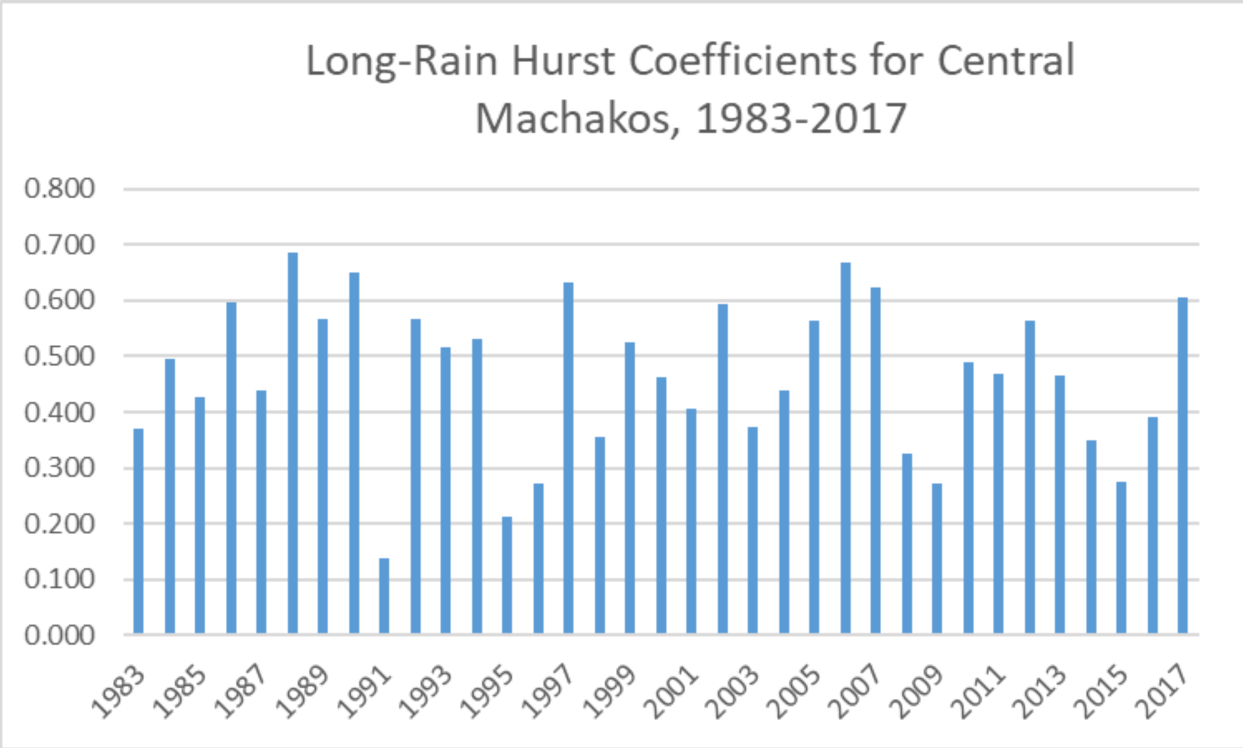


Figure 3

To investigate the properties of the distribution, we compute the Hurst coefficient for each year using the scaled variance method. The Hurst coefficient captures path dependency as being either ergodic, for H<0.5, or persistent for H>0.5. The argument here is that the deviations are not independent, but rather correlated or mean reverting for H<0.5 and positively reinforcing for H>0.5.


Long-Rain Hurst Coefficients for Central Machakos, 1983-2017
0.800
0.700
0.600
0.500
0.400
0.300
0.200
0.100
0.000
1983
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1987
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Figure 4

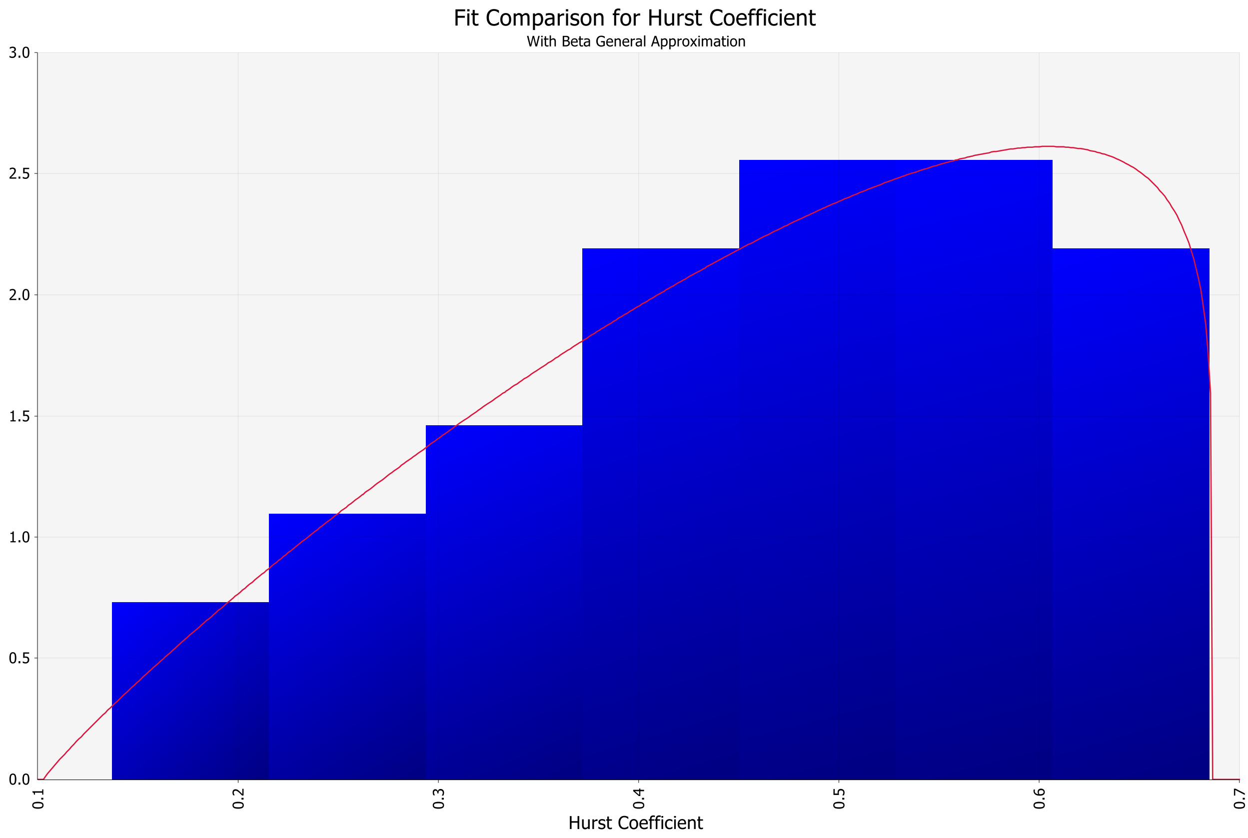


Figure 5

The historical distribution of these Hurst coefficients is provided in Figure 5 with the Beta General distribution approximation. The mean is H= 0.466 which, with a standard deviation of 0.137, is not statistically different from 0.5. The distribution is negatively skewed , S=-.4416 and slightly kurtosic at 2.58. The probability of H<0.5 is 57.1% and H>0.5 at 42.9%.

Figures 6 and 7 show the scatter plot and power functions for cumulative rainfall (RSquare = 0.1064) and damage intensity (RSquare = 0.1154) with the Hurst coefficients. The damage intensity is taken from our measure of indemnity using a dynamic trigger as discussed in more detail below, but for now it shows that a 1% increase in the Hurst coefficient corresponds on average with an increase in 'damage' of 0.717% (p=0.046). Similarly a 1% increase in the Hurst coefficient corresponds on average with an increase cumulative rainfall by 0.3573% (p=0.056). Although the overall fit of these regressions is low, the relationships are interesting. Generally speaking, drought intensity increases wit hlower Hurst coefficients. The characteristic of low Hurst coefficient is that the seasonal rainfall patterns are mean reverting; in otherwords te intertemporal covariance relationship is negative. Thus in drought years an increase in rainfall is more likely in probability to be followed by a a shortfall in rain. In contrast the high rainfall years with H>0.5 have a positive covariance suggesting that that increases in rainfall are reinforcing.

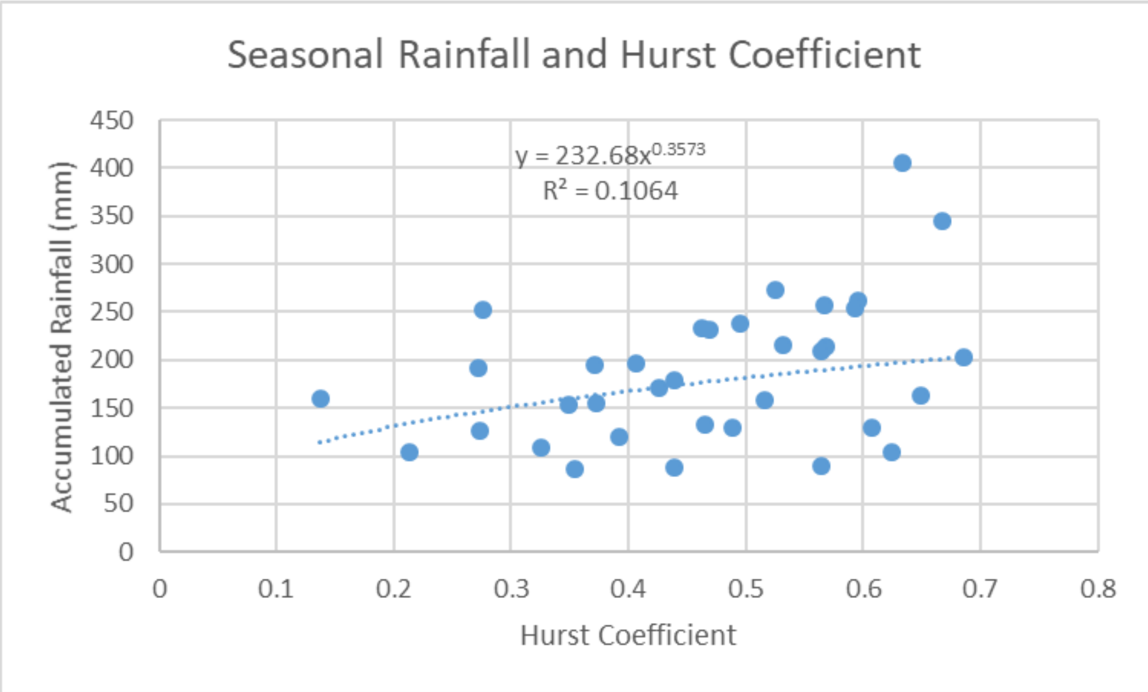


Figure 6

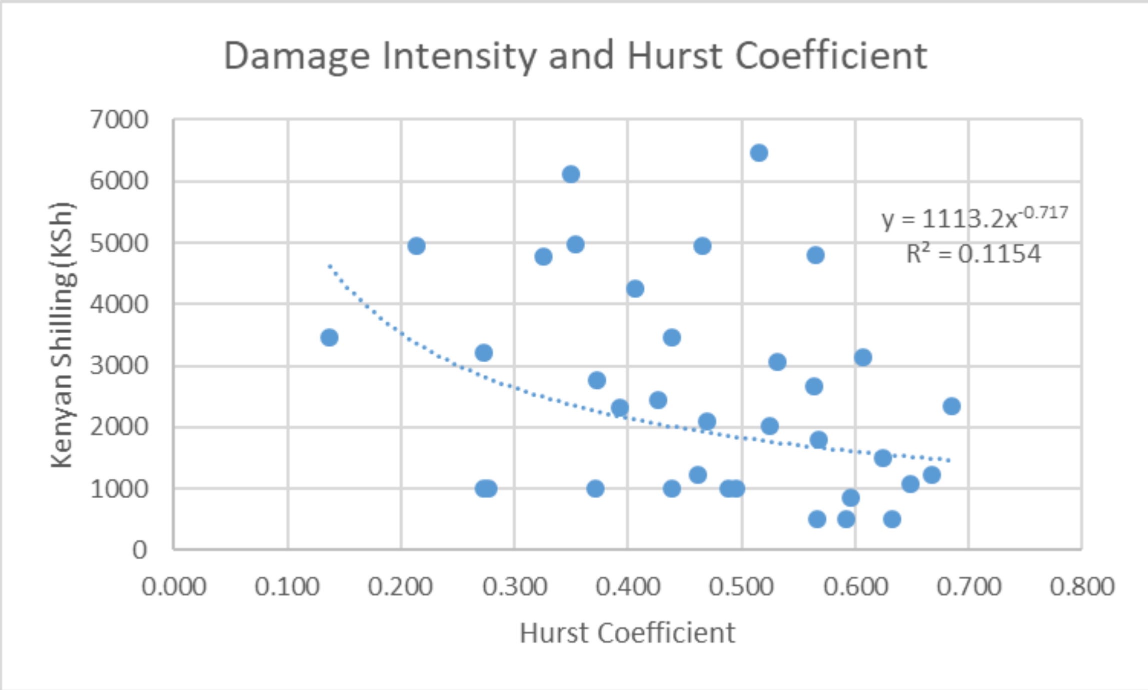


Figure 7

Resolving the Effects of Noah- and Joseph-Erratic Weather Patterns on Index Insurance with a Dynamic Trigger

Although we find strong evidence of Noah- and Joseph-erratic phenomenon in our subject area that does not imply that the weather risks are uninsurable. It does mean that in the traditional sense that simplified measures – including our own 2017-2018 cumulative rainfall model – will in many instances fail in the most basic efficiency measure of minimizing Type I (which is costly to the insurer) and Type II (which is costly to the farmer) error. Thus, there is a need to rethink insurance risks and indemnity structures

The backstory to this assessment is the results from a randomized control experiment to implement Risk-Contingent Credit (RCC) in 2017 to unbanked farmers in Machakos County Kenya (Shee and Turvey (2012), Marr et al (2016), von Negenborn et al 2018, Pelka et al (2015))[6]. The RCC product was in the form of a loan, which paid an indemnity if the accumulated

[6] Marr, Ana, Anne Winkel, Marcel van Asseldonk, Robert Lensink, Erwin Bulte, (2016) "Adoption and impact of index-insurance and credit for smallholder farmers in developing countries: A systematic review", Agricultural Finance Review, Vol. 76 Issue: 1, pp.94-118.

rainfall between October 15th and January 15th (the long rains) fell below a rainfall trigger established in millimeters (mm) below the 15th percentile, or for an event that might occur about once in every 7 years. The RCT design made approximately 350 offers of traditional credit, 350 offers of Risk-Contingent Credit (RCC), and as a control 350 turn-downs. Whether credit was received, and what terms were provided was by random selection. The simplicity of the design was intentional. The subject farmers had largely no interaction with formal banking services, let alone credit, and also had no experience with weather (or any type of crop-related) insurance. However, in pre-experiment focus groups with farmers across the Machakos district, it was clear that failures of the rain were the biggest risks faced, and lenders also acknowledged that failure of the rains was the largest impediment to providing agricultural credit. Although we were aware of erratic weather patterns it was felt by the research team, local bank, and local insurer that a simple design for a first-time pilot of a new bundled credit product would be the least complicated approach, with product modifications and scaling up to follow.

Weather conditions in 2017 were not satisfactory for the pilot. Depicted in the lower right panel of Figure 2, the rains failed between vegetative and flowering/maturity stages resulting in yield declines in excess of 50% for many borrower farmers. However, because of the mid-season rainfall the insurance did not trigger. Fortunately, we had anticipated that possibility and used reserve funds to provide an indemnity equal to 50% of loan balances for those receiving RCC, but no indemnity for those receiving traditional loans.

To address the erratic nature of rainfall patterns we propose a 21-day event model. By 'event' we refer to any 21 day period in the insured season in which accumulated rainfall falls below 60% of the average accumulated rainfall for that district over the same historical 21-day period (on a calendar basis, ignoring leap years). Generally speaking, and as depicted in Figure 1, the pattern of rainfall is low at the beginning of the season, rising, and then decreasing towards the end of the season on January 15th. The triggering event is dynamic, in the sense that it maps onto

Pelka, Niels, Oliver Musshoff, Ron Weber, (2015) "Does weather matter? How rainfall affects credit risk in agricultural microfinance", Agricultural Finance Review, Vol. 75 Issue: 2, pp.194-212

Sheet, A. and C.G. Turvey (2012) "Collateral-free Lending with Risk-Contingent Credit for Agricultural Development: Indemnifying Loans Against Pulse Crop Price Risk in India" Agricultural Economics 43:561-574

Shee, Apurba, Calum G. Turvey, Joshua Woodard, (2015) "A field study for assessing risk-contingent credit for Kenyan pastoralists and dairy farmers", Agricultural Finance Review, Vol. 75 Issue: 3, pp.330-348

von Negenborn, Freya, Ron Weber, Oliver Musshoff, (2018) "Explaining weather-related credit risk with evapotranspiration and precipitation indices", Agricultural Finance Review, Vol. 78 Issue: 2, pp.246-261

the historical rainfall pattern, rising and falling accordingly. The effects of a Dynamic Trigger are illustrated below for Central Machakos in 2015, with one small event, and 2017 with one small, and two significant events. The green line is cumulative rainfall, the blue line is the rolling 21-day cumulative rainfall, while the red dashed line is the Dynamic Trigger. Arrows indicate an ‘Event’, in which the rolling 21-day cumulative rainfall falls below the Dynamic Trigger. For example, the event horizon starts first at day 1- day 21, then day 2-day 22, day 3-day 23 until the end of the season day 73-day 93. In other words there are 73 consecutive 21-day periods in the long rain period between October 15 and January 15th. Each period is examined to determine if the actual rainfall was below the trigger. If not, then the next sequential 21-day period is examined and so on. If the actual rainfall is below the corresponding trigger, an event is triggered. Subsequent events cannot be overlapping. For example, if no event is recorded at day 21 or day 22, but is recorded for day 23, another event cannot be recorded on day 24. The next possible date for a second event would be day 44, covering the 21 days between day 24 and day 44. Since events cannot be overlapping, at most 4 events could be recorded in a single long rain season.

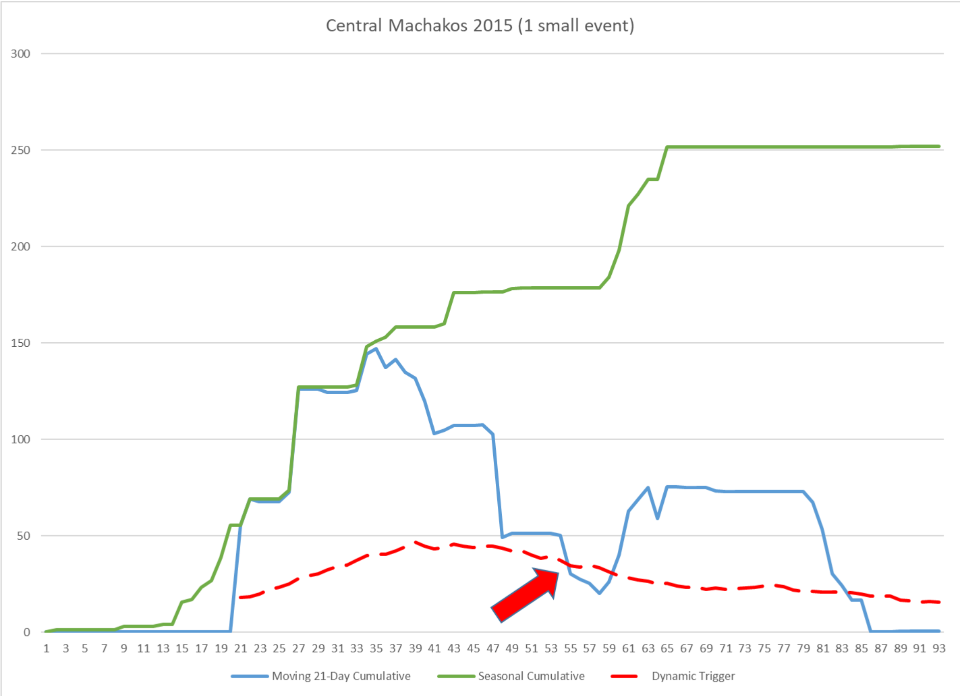


Figure 8

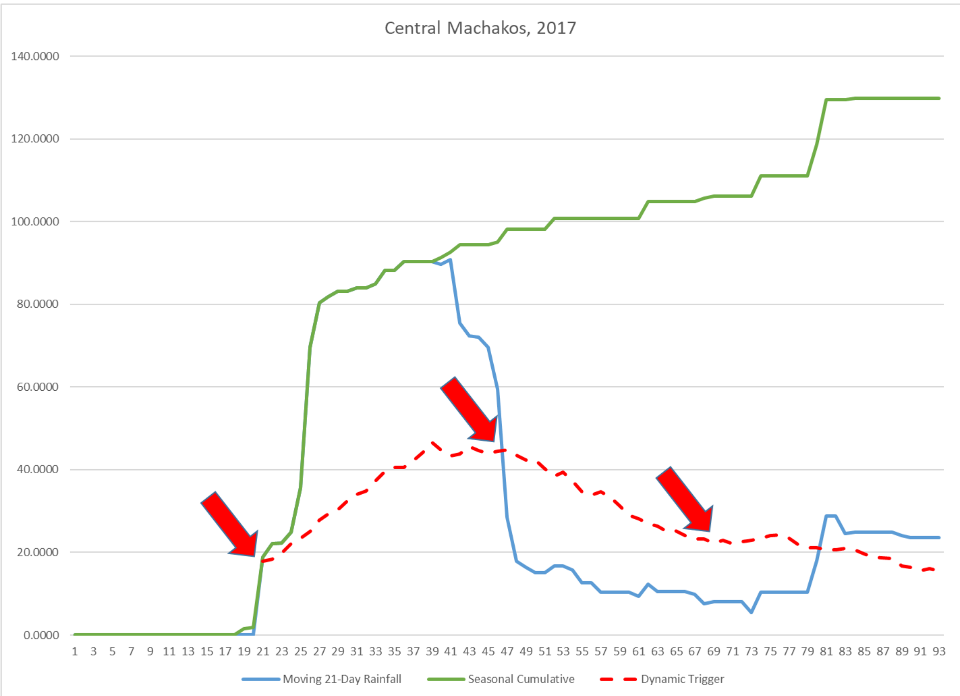


Figure 9

The rainfall deficit is measured by the difference between the rainfall trigger and actual rainfall in in millimeters (mm). Monetizing this requires multiplying the deficit (in mm units) by a nominal unit (KSh/mm) – the tick value - to obtain an indemnity for that event in KSh. In our modelling we assumed that if an event was triggered, the minimal indemnity for the event was to be 500 KSh.

The number of events for each year since 1983 for Central Machakos are provided in Figure 10 below, for a Dynamic Trigger based on 60% of cumulative rainfall for each 21-day period. The schema for the RCC design is provided in Figure 11, and the indemnity history for Central Machakos is provided in Figure 12.

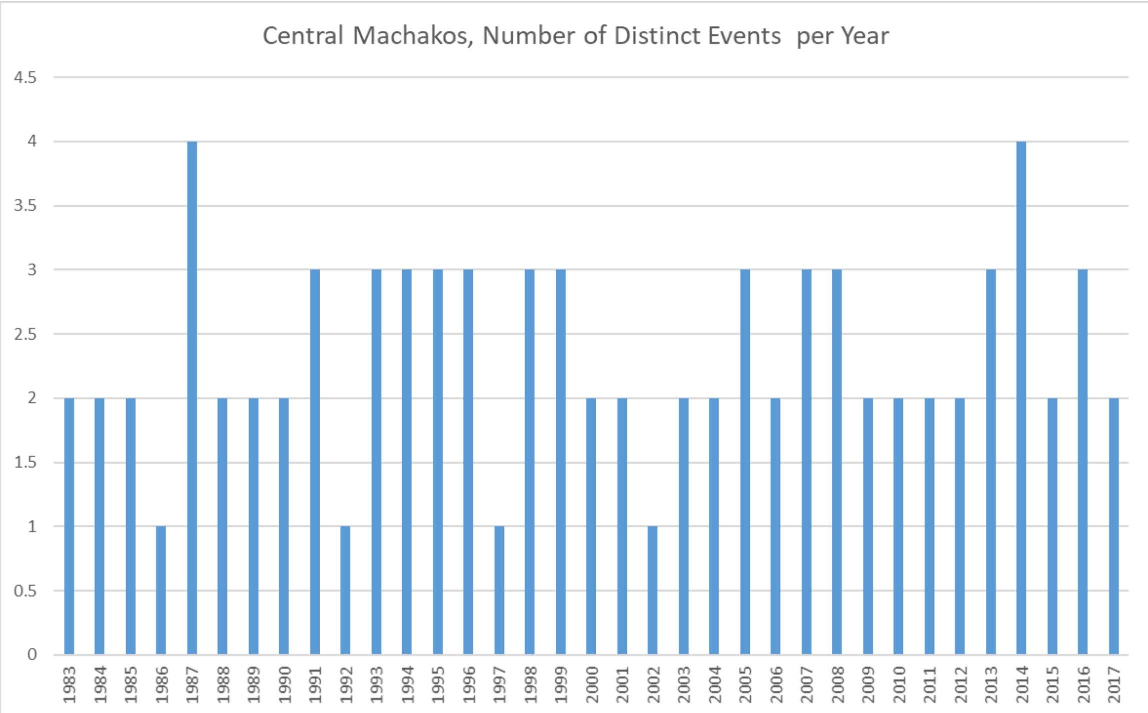


Figure 10

$$Z_s = Dynamic\,Trigger\ for\ any\ 21-day\ period\,(mm)$$

$$s = Rolling\ 21-day\ counter$$

$$R_s = Actual\ Long\ Rain\ Total\ for\ any\ 21-day\ period\,(mm)$$

$$R_s = \sum_{t=1}^{21} R_t \rightarrow R_1 = \sum_{t=1}^{21} R_t, R_2 = \sum_{t=2}^{22} R_t, \ldots R_{72} = \sum_{t=72}^{93} R_t$$

....

$$\psi = Tick\ Value\,(Ksh/mm)$$

$$If\ R_s < Z_s$$

$$Indemnity_s = \left(Z_s - R_s\right) \times \psi$$

$$Payout_s\,(Ksh) = Max\left(500, \left(Z_s - R_s\right) \times \psi\right)$$

$$Total\ Indemnity = \sum_{k=1}^{Max=4} Max\left(500, \left(Z_{s\rightarrow k} - R_{s\rightarrow k}\right) \times \psi\right)$$

$$Insurance\ Premium\ Rate = \theta$$

$$\theta = \frac{E\left[\sum_{k=1}^{Max=4} Max\left(500, \left(Z_{s\rightarrow k} - R_{s\rightarrow k}\right) \times \psi\right)\right]}{10,000}$$

....

$$f = Loan\ Principal\,(Ksh) = Loan\ Request + Insurance\ Premium = Loan\ Request \times \left(1 + \theta\right)$$

$$r = effective\ annual\ interest\ rate$$

$$T = time\ to\ loan\ repayment$$

$$Farmer\ Repayment = f\left(1 + \frac{r}{T}\right) - \sum_{k=1}^{Max=4} Max\left(500, \left(Z_{s\rightarrow k} - R_{s\rightarrow k}\right) \times \psi\right)$$

Figure 11: Model Schema for Design of Weather Index Insurance for a Bundled Risk-Contingent Credit Product

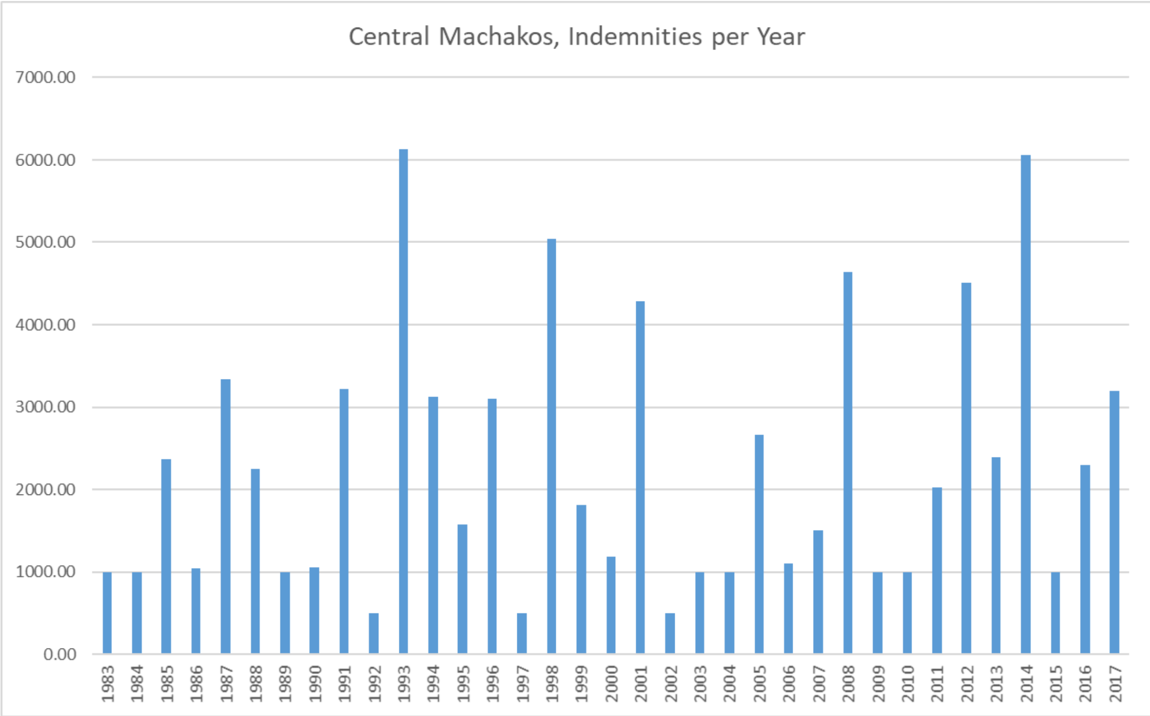
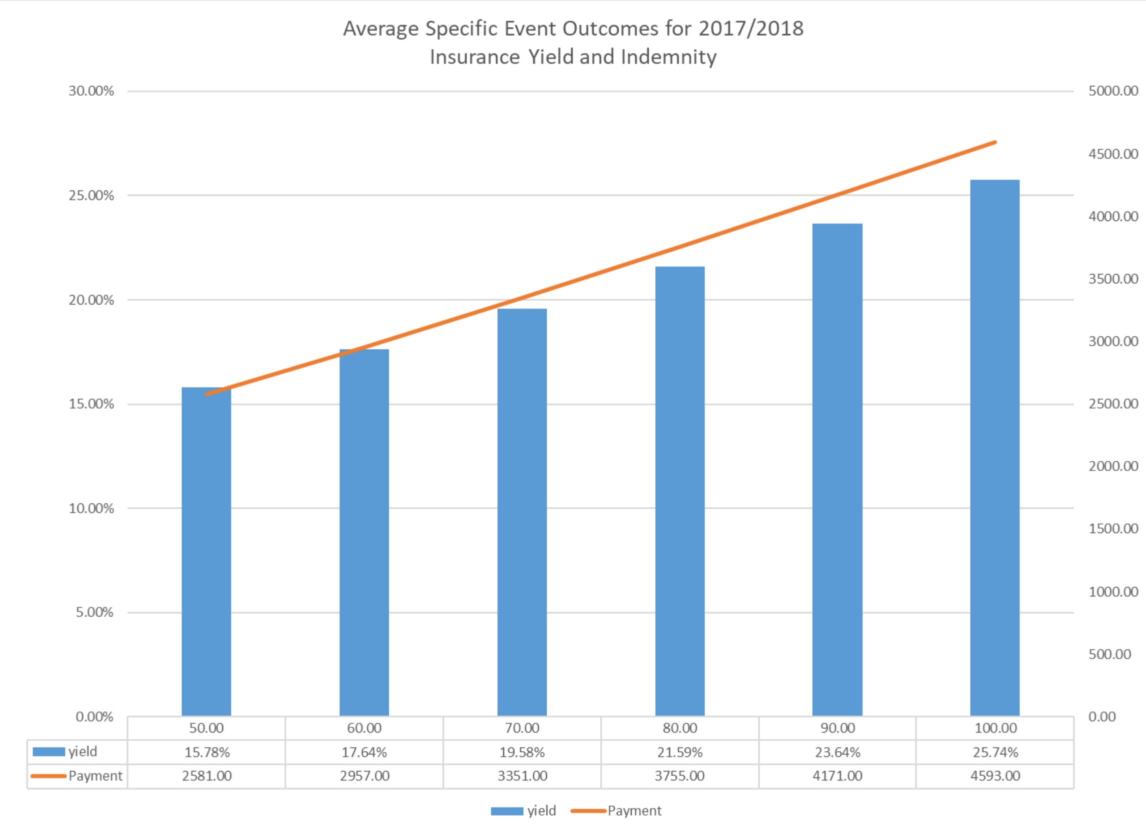


Figure 12

Model Variants

Ultimately, the final premium is based on two variables, which we can vary. The first variable is the trigger level, and the second is tick value. Figure 13 below illustrates the average premium rate and 2017/2018 indemnity based on a 10,000 KSh loan holding the Dynamic Trigger at 60% of average, and varying the tick value from 50 KSh/mm to 100 KSh/mm. At 50KSh/mm the insurance yield is 15.78%. This increases to 25.74% for a tick of 100 KSh/mm.



	50.00	60.00	70.00	80.00	90.00	100.00
yield	15.78%	17.64%	19.58%	21.59%	23.64%	25.74%
Payment	2581.00	2957.00	3351.00	3755.00	4171.00	4593.00

Figure 13

Concluding Remarks

In assessing the hydrology of water flows in rivers, Mandelbrot and Wallis (1968) established certain properties of stochastic processes that were erratic. The erratic nature of weather patterns, and particularly participation, complicates the design of weather index insurance products. In this paper we investigated the erratic nature of rainfall patterns in Machakos County, Kenya. The motivation for the paper was to ensure an optimal design of rainfall insurance that would minimize Type I and Type II error by reducing temporal basis risk. This is a key feature for implementation of risk-contingent credit.

Our findings point to an important warning sign for weather index insurance designe. We find that the patterns of rainfall are indeed erratic and consistent with the Noah and Joseph descriptors discussed by Mandelbrot and Wallis (1968). The erratic nature of rainfall emerges from two statistical failings. The first is a breakdown of the convergence to a normal distribution around the mean of our 21-day rainfall measure. Instead we find that the distribution about the average is approximately lognormal, with an almost 50% higher chance of deficit rainfall below the mean versus adequate rainfall above the mean. Perhaps more important is our finding that the rainfall patterns obey Hurst's law. We find that the Hurst coefficients for the average pathway is about H=0.8, but the range of Hurst coefficients across all years ranged from a low below H=0.2 and a high above H=0.6. The average Hurst, however was not significantly different from 0.5, but this is meaningless in an insurance context.

Because of the erratic nature of rainfall, we develop a new approach to weather index insurance based upon the accumulated rainfall in any 21-day period falling below 60% of the long term average for that same 21-day period. We argue that this approach is more satisfactory to matching drought conditions within and between various phenological stages of growth. While this new approach reduces Type I and Type II error, it comes at higher cost.